

# Learning Fair Policies in Multiobjective (Deep) Reinforcement Learning with Average and Discounted Rewards

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# Overview

- 1 Motivation and Problem
- 2 Theoretical Discussions & Algorithms
- 3 Experimental Results
- 4 Conclusion

# Motivation: Why should we care about fair systems?

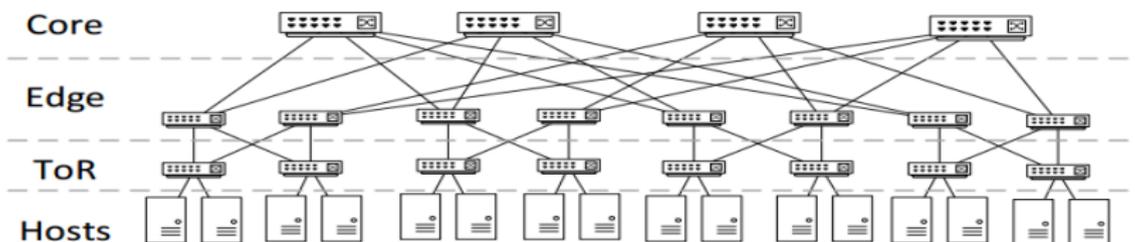


Figure: Network with a fat-tree topology from Ruffy et al. (2019).

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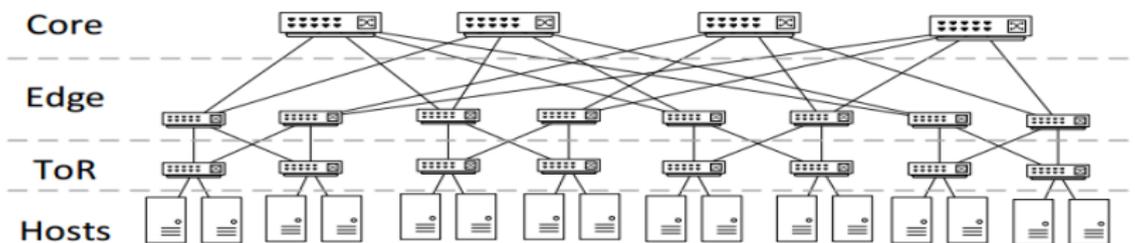


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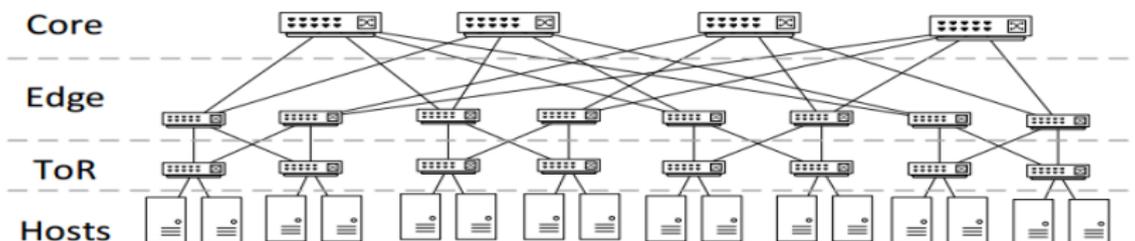


Figure: Network with a fat-tree topology from Ruffy et al. (2019).

- Fairness consideration to users is crucial
- Existing approaches to tackle this issue includes:
  - Utilitarian approach
  - Egalitarian approach

- Fairness includes:
  - Efficiency
  - Impartiality
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- We focus on *generalized Gini social welfare function* (GGF)

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$$\text{where } \mathbf{J}(\pi) = \mathbb{E}_{P_{\pi}} \left[ \sum_{t=1}^{\infty} \gamma^{t-1} \mathbf{R}_t \right] \quad \text{or} \quad \mathbf{J}(\pi) = \lim_{h \rightarrow \infty} \frac{1}{h} \mathbb{E}_{P_{\pi}} \left[ \sum_{t=1}^h \mathbf{R}_t \right].$$

$\gamma$ -discounted rewards                      average rewards

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**Theorem:**

$$\text{GGF}_{\mathbf{w}}(\boldsymbol{\mu}(\pi_\gamma^*)) \geq \text{GGF}_{\mathbf{w}}(\boldsymbol{\mu}(\pi_1^*)) - \bar{\mathbf{R}}(1 - \gamma) \left( \rho(\gamma, \sigma(\mathbf{H}_{\mathbf{P}_{\pi_1^*}})) + \rho(\gamma, \sigma(\mathbf{H}_{\mathbf{P}_{\pi_\gamma^*}})) \right)$$

where  $\bar{\mathbf{R}} = \max_{\pi} \|\mathbf{R}_{\pi}\|_1$  and  $\rho(\gamma, \sigma) = \frac{\sigma}{\gamma - (1 - \gamma)\sigma}$ .

# Value Based and Policy Gradient Algorithms

- DQN: Q network takes values in  $\mathbb{R}^{|\mathcal{A}| \times D}$ , instead of  $\mathbb{R}^{|\mathcal{A}|}$ , trained with target:

$$\hat{Q}_{\theta}(s, a) = \mathbf{r} + \gamma \hat{Q}_{\theta'}(s', a^*),$$

where  $a^* = \operatorname{argmax}_{a' \in \mathcal{A}} \operatorname{GGF}_{\mathbf{w}}(\mathbf{r} + \gamma \hat{Q}_{\theta'}(s', a'))$ .

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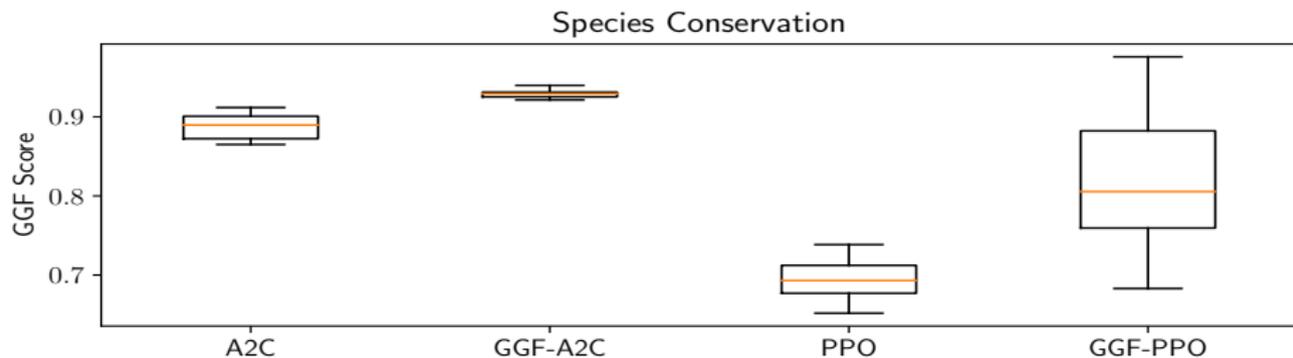
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- To optimize the GGF with policy gradient:

$$\begin{aligned} \nabla_\theta \operatorname{GGF}_w(\mathbf{J}(\pi_\theta)) &= \nabla_{\mathbf{J}(\pi_\theta)} \operatorname{GGF}_w(\mathbf{J}(\pi_\theta)) \cdot \nabla_\theta \mathbf{J}(\pi_\theta) \\ &= \mathbf{w}_\sigma^\top \cdot \nabla_\theta \mathbf{J}(\pi_\theta). \end{aligned}$$

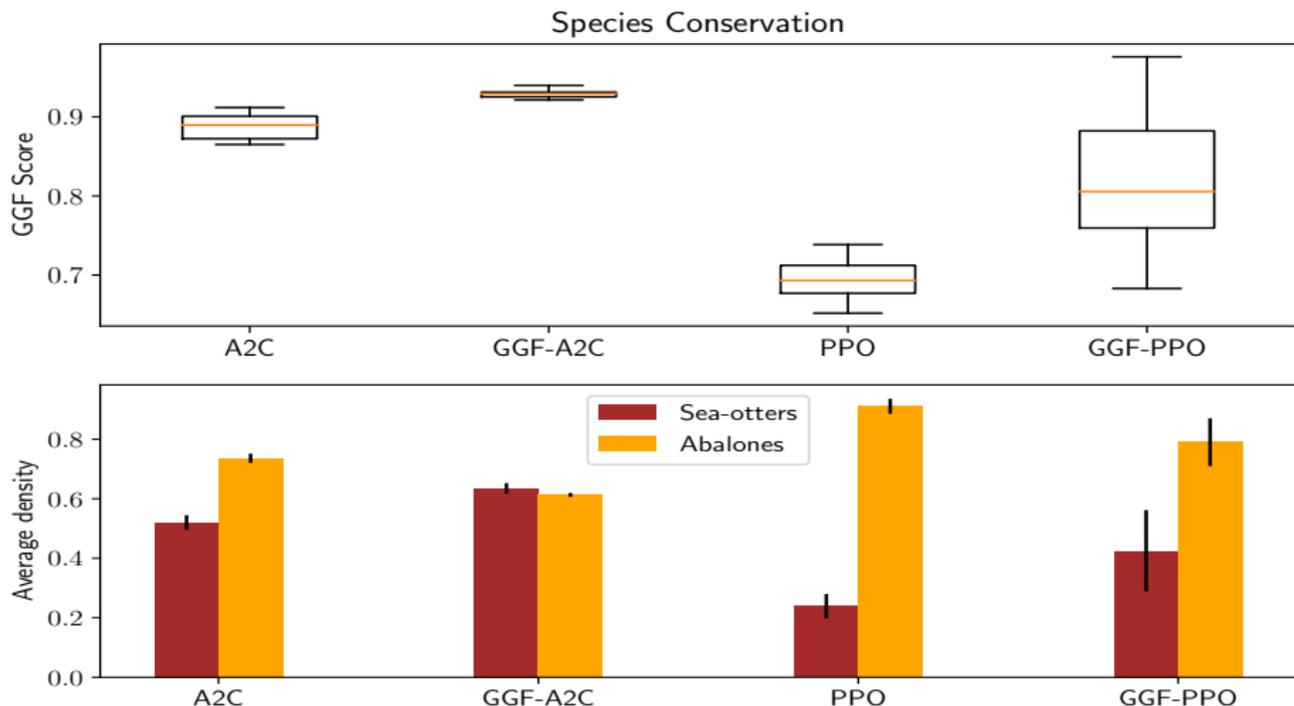
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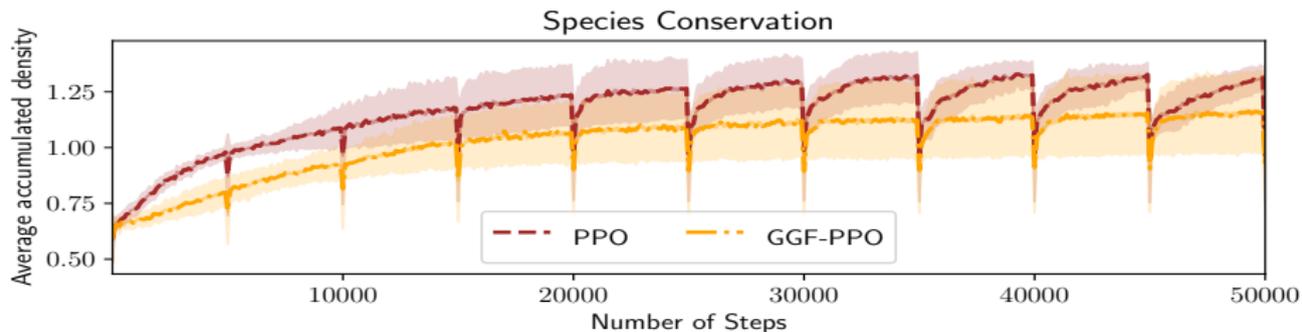
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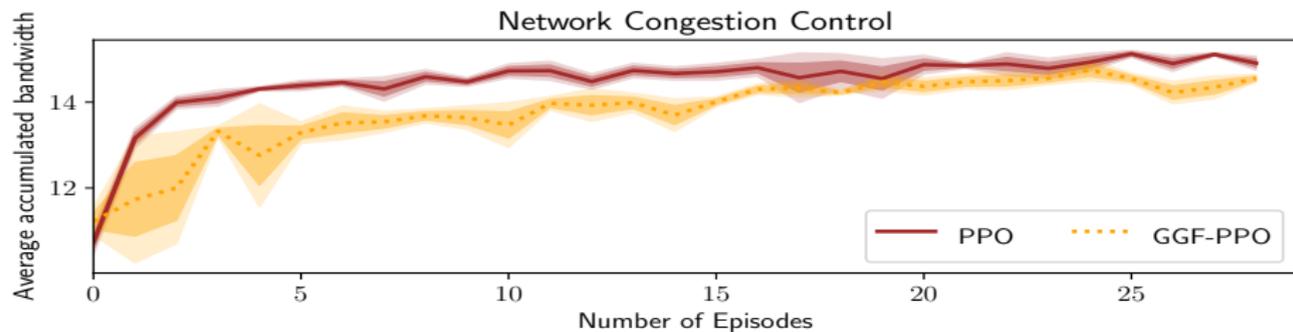
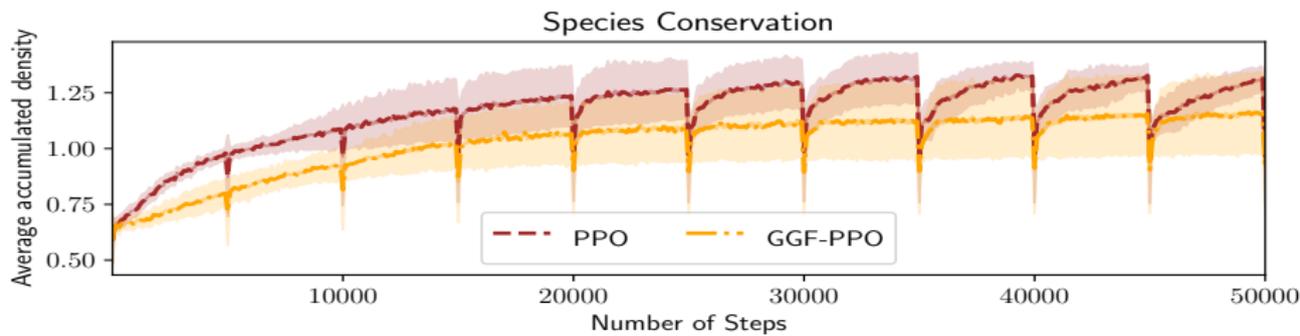
How those algorithms performs in continuous domains?



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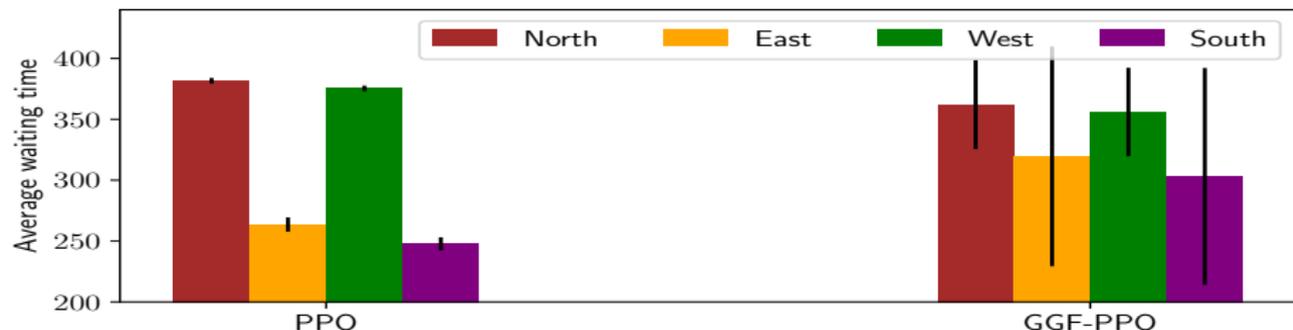
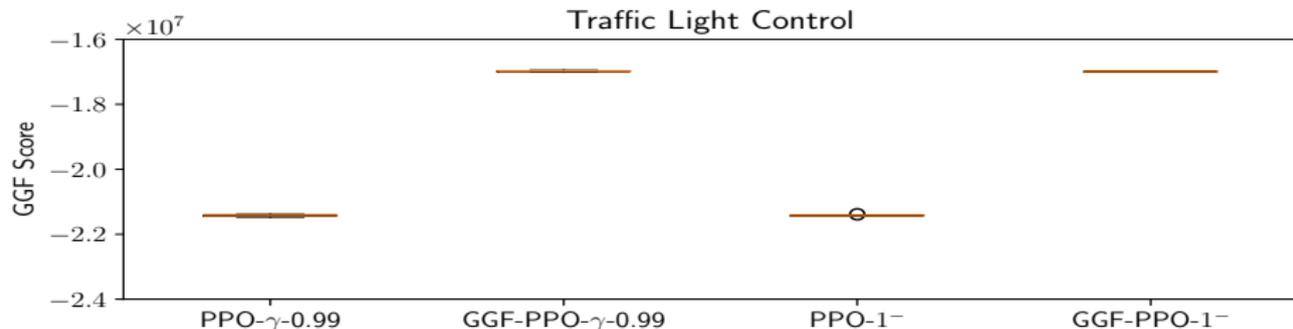
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# Experimental Results (Traffic Light Control)

What is the effect of  $\gamma$  with respect to GGF-average optimality?



# Conclusion

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## Future Works:

- Extend to distributed control
- Consider other fair social welfare functions
- Directly solve average reward problems

Ruffy, F., Przystupa, M., and Beschastnikh, I. (2019). Iroko: A framework to prototype reinforcement learning for data center traffic control. In *Workshop on ML for Systems at NeurIPS*.