Learnable Group Transform for Time-Series

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Challenges in Time-Series Example

Dataset[1]**:** *Audio field recordings* **Task:** *Binary classification*

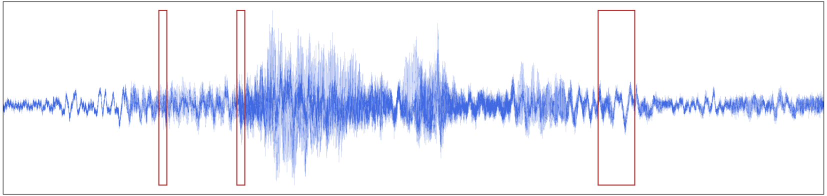


Figure: Dimension: $440,000$. The red boxes are the locations of the bird song.

Several Challenges:

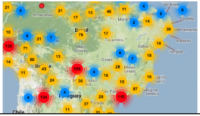
- High-dimensional signals.
- Large intra-class variability.
- A lot of nuisances.

[1] http://machine-listening.eecs.qmul.ac.uk/bird-audio-detection-challenge/

Same Challenges Across Domains

Toulon univ. DYNI LIS SMIoT

Measuring station in Malmo, Dalaplan



Xeno canto Audio records centered on Brazil

Various Domains

- Biodiversity monitoring
- Speech Recognition
- Health Care
- Earth Sciences

Common Approach To Overcome These Challenges

1. Project the data in the Time-Frequency plane
2. Use this Time-Frequency representation as the input of a Deep Neural Network

We focus on the Time-Frequency Representation

Time-Frequency Representation: Example

- Time-Frequency representations, e.g.: Wavelet transform, Short-time Fourier transform, Deep Scattering Network, Mel Frequency Cepstral Coefficients.

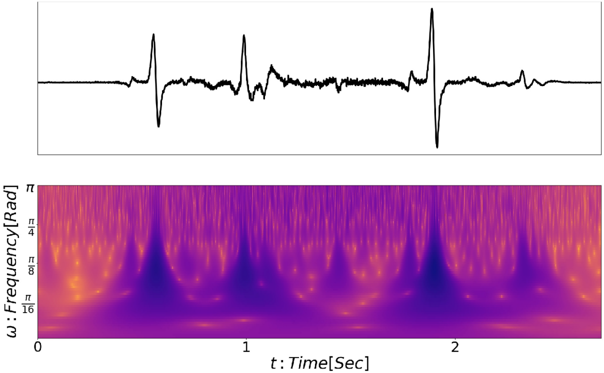


Figure: Dimension: 2,500. Intra Cardiac Recording.

Intrinsic Problems of Hand-crafted Time-Frequency Representations:

- Often not aligned with the task: Clustering, Prediction, Classification, ...
- Require expert knowledge on the data and the task.
- Require cross-validation of parameters s.a.: number of octaves and wavelets per octave, size of the window,...
- Such knowledge may not exist. Example: prediction of seismic activity, seizure prediction.

We propose a data-driven (end-to-end) approach

Building Blocks of Time-Frequency Representations

To obtain the Time-Frequency Representation of a signal

1. Build a specific Time-Frequency filter bank.
2. Convolve the filters with the signal.

Require Two Components to Create a Filter Bank

1. Select a mother filter ψ.
2. Select a transformation space $\mathcal{F}$.

$$\text{Filter Bank} = \{\psi \circ g_1, \ldots, \psi \circ g_K | g_1, \ldots, g_K \in \mathcal{F}\}.$$

The g_k are samples from the space $\mathcal{F}$.

Convolution Between Filters and Signal Equals Time-Frequency Representation

Given a signal by s, its Time-Frequency representation is given by

$$\text{Time-Frequency Representation} = [\mathcal{W}[s,\psi](g_1,.),\ldots,\mathcal{W}[s,\psi](g_K,.)]^T,$$

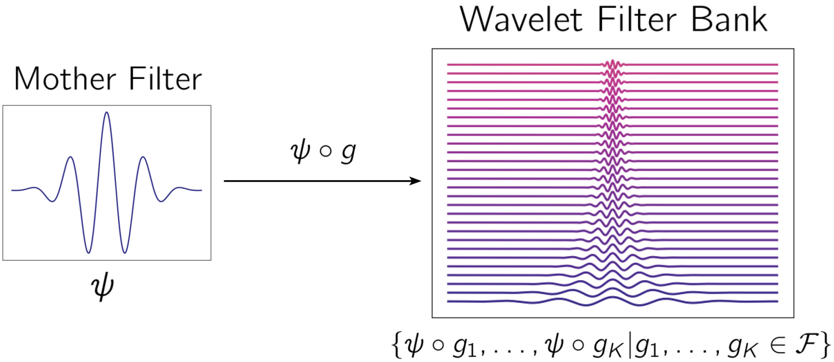
where

$$\mathcal{W}[s,\psi](g,.) = s \star (\psi \circ g), \forall g \in \mathcal{F},$$

with $\star$ the convolution operator and (.) corresponds to the time axis.

Filter Bank Example: Wavelet Filter Bank

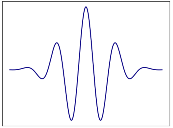
Mother Filter ψ: Morlet Wavelet **Transformation Space** $\mathcal{F}$: Linear $g(t) = \frac{t}{\lambda}$



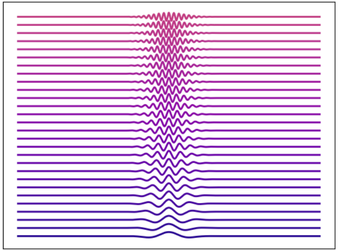
We propose to focus on the learnability of the Transformation Space $\mathcal{F}$.

Different Transformation Space For the Same Mother Filter

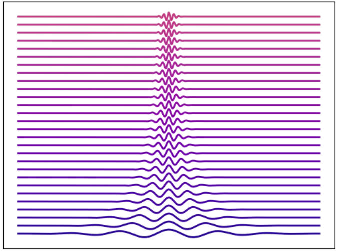
Mother Filter



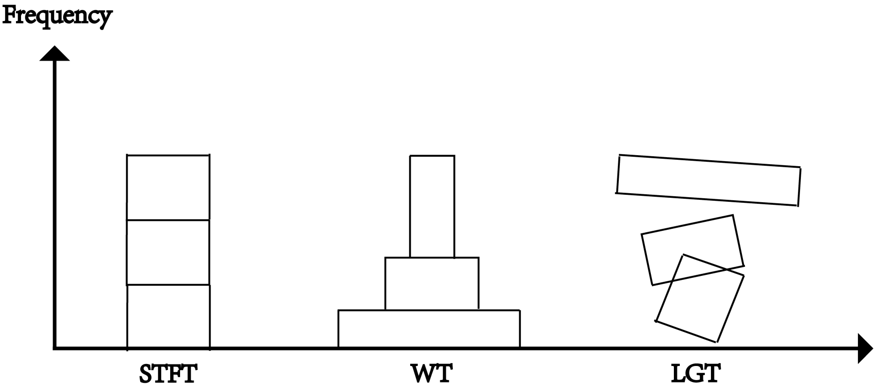
STFT Filters Bank



Wavelet Filters Bank



Transformation Space Induces the Tiling of the Time-Frequency Plane



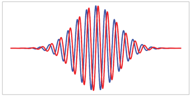
- Different Transformation Space ⇒ different Time-Frequency Resolutions.
- All are constrained by the Heisenberg uncertainty principle.

The Space of Continuous and Strictly Increasing Functions

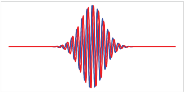
A direct generalization of the **Transformation Space** of Wavelet Filter Bank is given by

$$C^0_{\text{inc}}(\mathbb{R}) = \left\{ g \in C^0(\mathbb{R}) | g \text{ is strictly increasing} \right\},$$

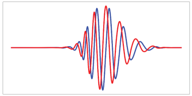
where $C^0(\mathbb{R})$ defines the space of continuous functions defined on $\mathbb{R}$.



Mother Filter



Affine Transformation



Non-Linear Transformation

Recovering well-known filters From $C^0_{\text{inc}}(\mathbb{R})$

$g \in C^0_{inc}(\mathbb{R})$	$\psi \circ g$
Affine	Wavelet
Quadratic Convex	Increasing Quadratic Chirplet
Quadratic Concave	Decreasing Quadratic Chirplet
Logarithmic	Logarithmic Chirplet
Exponential	Exponential Chirplet

Sampling and Learning $g \in C^0_{\text{inc}}(\mathbb{R})$

1. **Sampling:**

Strictly Increasing Piecewise Continuous Functions can be re-written as a 1-layer ReLU Neural Network.

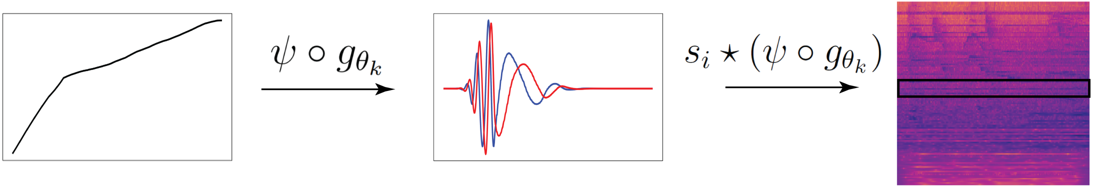
2. **Learning:**

Given a set of signals $\{s_i\}_{i=1}^N$, a mother Filter ψ, a Deep Neural Network F designed for a specific task represented by the loss L,

$$\min_{\Theta} \sum_{i=1}^{N} L\big(F(\mathcal{W}[s_i, \psi](\mathbf{g}_{\Theta}, .))\big),$$

where Θ are the parameters of the 1-layer ReLU Neural Network.

Learnable Group Transform: Framework



1. Sample g_{θ_k} From 1-Layer ReLU NN.
2. Compose the Mother Wavelet ψ with g_{θ_k}.
3. Convolve $\psi \circ g_{\theta_k}$ with signal s_i.

Experiments

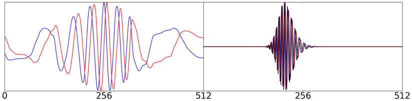
Evaluation of our method on three datasets:

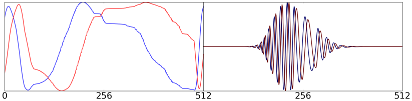
1. Artificial Data: Increasing Chirp VS Decreasing Chirp.
2. Haptics Data: Small dataset where the optimal Time-Frequency Representation is unknown.
3. Bird Song Classification: Large Scale dataset where the optimal Time-Frequency Representation is known.

We obtain results at the level of state of the art methods.

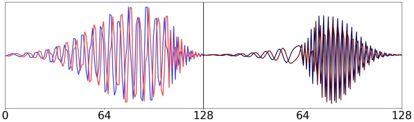
Learnable Group Transform Filters - Filter Analysis

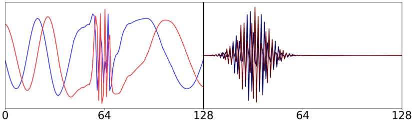
Samples of Learned Filters For Bird Song Dataset Classification Task:





Samples of Learned Filters For Haptics Dataset Classification Task:





Conclusion

- We propose an end-to-end approach to filter bank learning.
- Our approach generalize Wavelet Transform by proposing a **non-linear strictly increasing** transformation function as opposed to the **linear** one.
- Competes with state of the art methods in different applications.
- Recover optimal filters for Bird Song classification task.