# Scalable Differential Privacy with Certified Robustness in Adversarial Learning

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#### Outline

- Motivation and Background
- Differential Privacy (DP) in Adversarial Learning
- Composition of Certified Robustness
- Stochastic Batch Training (StoBatch)
- Experimental Results and Conclusion



#### Motivation

- DNNs are vulnerable to both privacy attacks and adversarial examples
- Existing efforts only focus on either preserving DP or deriving certified robustness, but not both DP and robustness!
  - private models are unshielded under adversarial examples
  - robust models (adversarial training) do not offer privacy protections to the training data

- Bounding the robustness of a model (protects data privacy and is robust against adversarial examples) at scale is nontrivial
  - adversarial examples introduces a previously unknown privacy risk
  - unrevealed interplay (trade-off) among DP preservation, adversarial learning, and robustness bounds



#### Goals

• Develop a novel mechanism (StoBatch) to: 1) preserve DP of the training data, 2) be provably and practically robust to adversarial examples, 3) retain high model utility, and 4) be scalable.

#### **Methods**

- Privacy-preserving (Laplace) noise is injected into inputs and hidden layers to achieve DP in learning private model parameters.
- The privacy noise p is projected on the scale of the robustness noise r.
- a composition of certified robustness in both input and latent spaces
- Leverage the recipe of distributed adversarial training to develop a stochastic batch training
- disjoint and fixed batches are distributed to local DP trainers

#### **Results**

- Established a connection among DP preservation to protect the training data, adversarial learning, and certified robustness.
- Derived a sequential composition robustness in both input and latent spaces.
- Addressed the trade-off among model utility, privacy loss, and robustness.
- Rigorous experiments shown that our mechanism significantly enhances the robustness and scalability of DP DNNs.

#### **Deliverables**

 Algorithms and models: <a href="https://github.com/haiphanNJIT/StoBatch">https://github.com/haiphanNJIT/StoBatch</a>

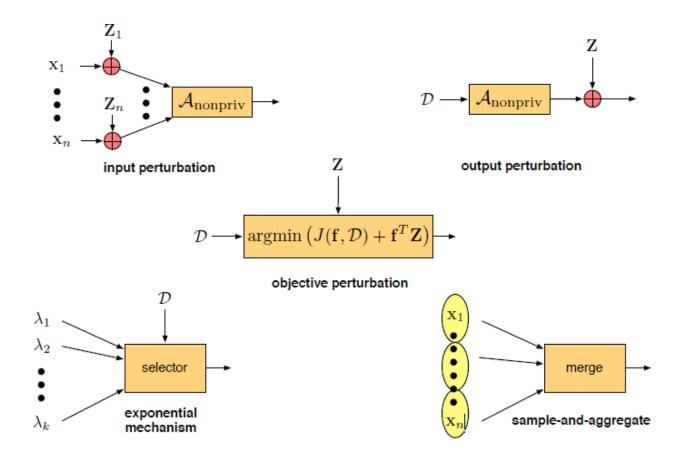
# Differential Privacy

 Databases D and D' are neighbors if they are different in one individual's contribution

•  $(\epsilon, \delta)$ -Differential Privacy: for all D, D' neighbors, the distribution of A(D) is (nearly) the same as the distribution of A(D') for all  $\mathbf{o}$ :

$$Pr[A(D) = \mathbf{o}] \leq e \Pr[A(D') = \mathbf{o}] + \delta$$
 privacy loss

### **DP** Mechanisms



[Chaudhuri & Sarwate]



# Robustness Condition [Lécuyer et al., 2019]

$$\forall \alpha \in l_p(\mu): f_k(x + \alpha) > \max_{i:i \neq k} f_i(x + \alpha)$$

where k = y(x), indicating that a small perturbation in the input does not change the predicted label y(x).



### DP with Certified Robustness

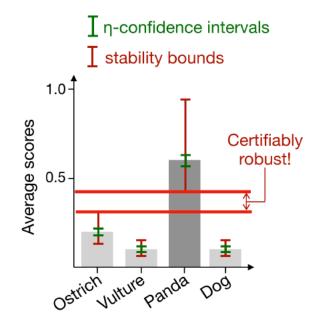
[Lécuyer et al., 2019]

• Image level:  $x = x + N(0, \sigma_r^2)$ 

• 
$$\sigma_r \ge \sqrt{2 \ln \left(\frac{1.25}{\delta_r}\right)} \Delta_r / \epsilon_r$$

$$\forall \alpha \in l_p(\mu = 1) : \hat{\mathbb{E}}_{lb} f_k(x) > e^{2\epsilon_r} \max_{i:i \neq k} \hat{\mathbb{E}}_{ub} f_i(x) + (1 + e^{\epsilon_r}) \delta_r$$

where  $\hat{\mathbb{E}}_{lb}$  and  $\hat{\mathbb{E}}_{ub}$  are the lower bound and upper bound of the expected value  $\hat{\mathbb{E}}f(x) = \frac{1}{N} \sum_{N} f(x)_{N}$ , derived from the Monte Carlo estimation with an  $\eta$ -confidence, given N is the number of invocations of f(x) with independent draws in the noise  $\sigma_r$ .



Robustness Test Example



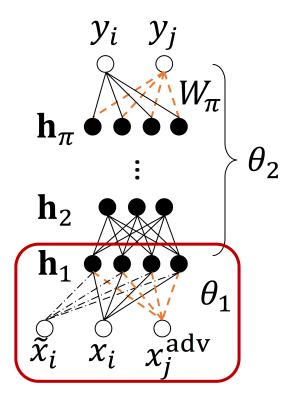
#### Outline

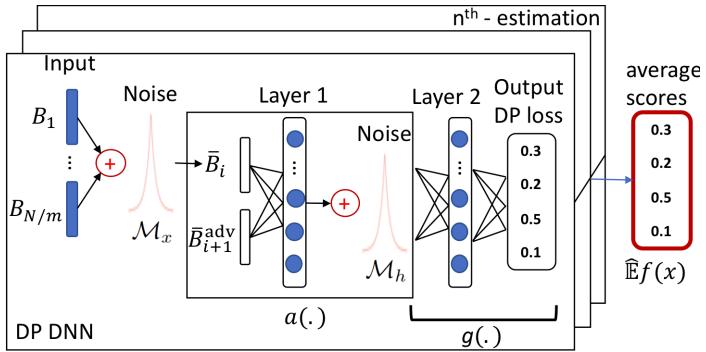
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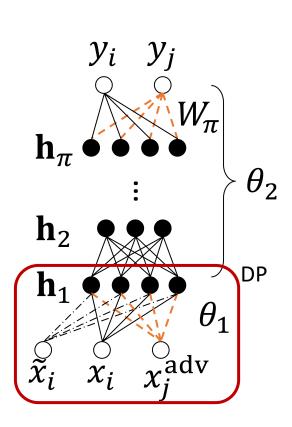
### Differential Privacy in Adversarial Learning [Overview]

- $f(x) = g(a(x, \theta_1), \theta_2)$ 
  - easier to train, small sensitivity bounds, and reusability





### DP Auto-Encoder



$$\bar{\mathcal{R}}_{\bar{B}_t}(\theta_1) = \sum_{x_i \in \bar{B}_t} \left[ \sum_{j=1}^d \left( \frac{1}{2} \theta_{1j} \bar{h}_i \right) - \bar{x}_i \tilde{x}_i \right]$$

$$\bar{x}_i = x_i + \frac{1}{m} Lap\left(\frac{\Delta_{\mathcal{R}}}{\varepsilon_1}\right)$$
, and  $\bar{h}_i = \theta_1^T \bar{x}_i + \frac{2}{m} Lap\left(\frac{\Delta_{\mathcal{R}}}{\varepsilon_1}\right)$ 

**Theorem 1** The gradient descent-based optimization of  $\overline{\mathcal{R}}_{\overline{B}_t}(\theta_1)$  preserves  $(\epsilon_1/\gamma_{\mathbf{x}} + \epsilon_1)$ -DP in learning  $\theta_1$ .

**Lemma 2** The global sensitivity of  $\widetilde{\mathcal{R}}$  over any two neighboring batches,  $B_t$  and  $B'_t$ , is:  $\Delta_{\mathcal{R}} \leq d(\beta + 2)$ .

# Adversarial Learning with DP

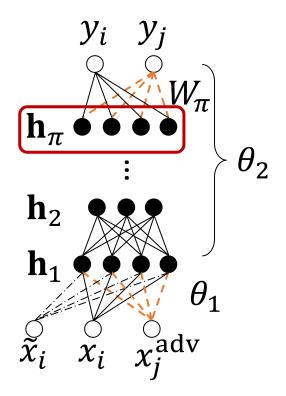
**Lemma 3** The computation of the batch  $B_t$  as the input layer is  $(\epsilon_1/\gamma_{\mathbf{x}})$ -DP, and the computation of the affine transformation  $\overline{\mathbf{h}}_{1\overline{B}_t}$  is  $(\epsilon_1/\gamma)$ -DP.

#### DP Adversarial Examples

$$\overline{x}_{j}^{\text{adv}} = \overline{x}_{j} + \mu \cdot \text{sign}\Big(\nabla_{\overline{x}_{j}} \mathcal{L}\big(f(\overline{x}_{j}, \theta), y(\overline{x}_{j})\big)\Big)$$

DP Objective function

as: 
$$\mathcal{L}_{\overline{B}_t}(\theta_2) \cong \sum_{k=1}^K \sum_{\overline{x}_i} \left[ \mathbf{h}_{\pi i} W_{\pi k} - (\mathbf{h}_{\pi i} W_{\pi k}) y_{ik} - \frac{1}{2} |\mathbf{h}_{\pi i} W_{\pi k}| + \frac{1}{8} (\mathbf{h}_{\pi i} W_{\pi k})^2 \right] \cong \mathcal{L}_{1\overline{B}_t}(\theta_2) - \mathcal{L}_{2\overline{B}_t}(\theta_2),$$
 where  $\mathcal{L}_{1\overline{B}_t}(\theta_2) = \sum_{k=1}^K \sum_{\overline{x}_i} \left[ \mathbf{h}_{\pi i} W_{\pi k} - \frac{1}{2} |\mathbf{h}_{\pi i} W_{\pi k}| + \frac{1}{8} (\mathbf{h}_{\pi i} W_{\pi k})^2 \right],$  and  $\mathcal{L}_{2\overline{B}_t}(\theta_2) = \sum_{k=1}^K \sum_{\overline{x}_i} (\mathbf{h}_{\pi i} y_{ik}) W_{\pi k}.$ 





#### **Algorithm 1 Adversarial Learning with DP**

**Input:** Database D, loss function L, parameters  $\theta$ , batch size m, learning rate  $\varrho_t$ , privacy budgets:  $\epsilon_1$  and  $\epsilon_2$ , robustness parameters:  $\epsilon_r$ ,  $\Delta_r^x$ , and  $\Delta_r^h$ , adversarial attack size  $\mu_a$ , the number of invocations n, ensemble attacks A, parameters  $\psi$  and  $\xi$ , and the size  $|\mathbf{h}_{\pi}|$  of  $\mathbf{h}_{\pi}$ 

- 1: **<u>Draw Noise</u>**  $\chi_1 \leftarrow [Lap(\frac{\Delta_{\mathcal{R}}}{\epsilon_1})]^d, \chi_2 \leftarrow [Lap(\frac{\Delta_{\mathcal{R}}}{\epsilon_1})]^{\beta}, \chi_3 \leftarrow [Lap(\frac{\Delta_{\mathcal{L}2}}{\epsilon_2})]^{|\mathbf{h}_{\pi}|}$
- 2: **Randomly Initialize**  $\theta = \{\theta_1, \theta_2\}$ ,  $\mathbf{B} = \{B_1, \dots, B_{N/m}\}$  s.t.  $\forall B \in \mathbf{B} : B$  is a batch with the size  $m, B_1 \cap \dots \cap B_{N/m} = \emptyset$ , and  $B_1 \cup \dots \cup B_{N/m} = D$ ,  $\overline{\mathbf{B}} = \{\overline{B}_1, \dots, \overline{B}_{N/m}\}$  where  $\forall i \in [1, N/m] : \overline{B}_i = \{\overline{x} \leftarrow x + \frac{\chi_1}{m}\}_{x \in B_i}$
- 3: Construct a deep network f with hidden layers  $\{\mathbf{h}_1 + \frac{2\chi_2}{m}, \dots, \mathbf{h}_{\pi}\}$ , where  $\mathbf{h}_{\pi}$  is the last hidden layer
- 4: for  $t \in [T]$  do
- 5: **Take** a batch  $\overline{B}_i \in \overline{\mathbf{B}}$  where  $i = t\%(N/m), \overline{B}_t \leftarrow \overline{B}_i$
- 6: Ensemble DP Adversarial Examples:
- 7: **Draw Random Perturbation Value**  $\mu_t \in (0, 1]$
- 8: **Take** a batch  $\overline{B}_{i+1} \in \overline{\mathbf{B}}$ , **Assign**  $\overline{B}_t^{\mathrm{adv}} \leftarrow \emptyset$
- 9: **for**  $l \in A$  **do**
- 10: **Take** the next batch  $\overline{B}_a \subset \overline{B}_{i+1}$  with the size m/|A|
- 11:  $\forall \overline{x}_j \in \overline{B}_a$ : **Craft**  $\overline{x}_j^{\text{adv}}$  by using attack algorithm A[l] with  $l_{\infty}(\mu_t)$ ,  $\overline{B}_t^{\text{adv}} \leftarrow \overline{B}_t^{\text{adv}} \cup \overline{x}_j^{\text{adv}}$
- 12: **<u>Descent:</u>**  $\theta_1 \leftarrow \theta_1 \varrho_t \nabla_{\theta_1} \overline{\mathcal{R}}_{\overline{B}_t \cup \overline{B}_t^{\text{adv}}}(\theta_1); \ \theta_2 \leftarrow \theta_2 \varrho_t \nabla_{\theta_2} \overline{L}_{\overline{B}_t \cup \overline{B}_t^{\text{adv}}}(\theta_2) \text{ with the noise } \frac{\chi_3}{m}$

**Output:**  $\epsilon = (\epsilon_1 + \epsilon_1/\gamma_{\mathbf{x}} + \epsilon_1/\gamma + \epsilon_2)$ -DP parameters  $\theta = \{\theta_1, \theta_2\}$ , robust model with an  $\epsilon_r$  budget

### Algorithm

$$L_{\overline{B}_t \cup \overline{B}_t^{\text{adv}}}(\theta_2) = \frac{1}{m(1+\xi)} \left( \sum_{\overline{x}_i \in \overline{B}_t} \mathcal{L}(f(\overline{x}_i, \theta_2), y_i) \right) + \xi \sum_{\overline{x}_j^{\text{adv}} \in \overline{B}_t^{\text{adv}}} \Upsilon(f(\overline{x}_j^{\text{adv}}, \theta_2), y_j) \right)$$

**Theorem 4** Algorithm 1 achieves  $(\epsilon_1 + \epsilon_1/\gamma_{\mathbf{x}} + \epsilon_1/\gamma + \epsilon_2)$ -DP parameters  $\overline{\theta} = {\overline{\theta}_1, \overline{\theta}_2}$  on the private training data D across T gradient descent-based training steps.



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# Composition of Certified Robustness

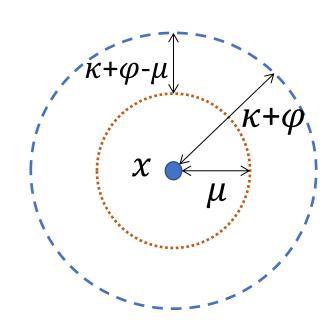
• Project the privacy noise p on the scale of the robustness noise r.

$$\kappa = \frac{\Delta_{\mathcal{R}}}{m\varepsilon_{1}} / \frac{\Delta_{r}^{x}}{\varepsilon_{r}}, \qquad \bar{x}_{i} = x_{i} + Lap\left(\frac{\kappa \Delta_{r}^{x}}{\varepsilon_{r}}\right)$$

$$\varphi = \frac{\Delta_{\mathcal{R}}}{m\varepsilon_{1}} / \frac{\Delta_{r}^{h}}{\varepsilon_{r}}, \qquad \bar{h}_{i} = h_{i} + Lap\left(\frac{\varphi \Delta_{r}^{h}}{\varepsilon_{r}}\right)$$

• What is the general robustness bound, given  $\kappa$  and  $\varphi$ ?

$$f(\mathcal{M}_1, \dots, \mathcal{M}_S | x) : \mathbb{R}^d \to \prod_{s \in [1, S]} f^s(x) \in \mathbb{R}^K$$



Sequential Composition of Certified Robustness: Lemma 5, Theorem 5

$$\forall \alpha \in l_p(\kappa + \varphi) : \hat{\mathbb{E}} f_k(x + \alpha) > \max_{i:i \neq k} \hat{\mathbb{E}} f_i(x + \alpha)$$



### Verified Inference

#### StoBatch Robustness

$$(\kappa + \varphi)_{max} = \max_{\epsilon_r} \frac{\Delta_{\mathcal{R}} \epsilon_r}{m \epsilon_1} (\frac{1}{\Delta_r^x} + \frac{2}{\Delta_r^h}) \text{ s.t.}$$

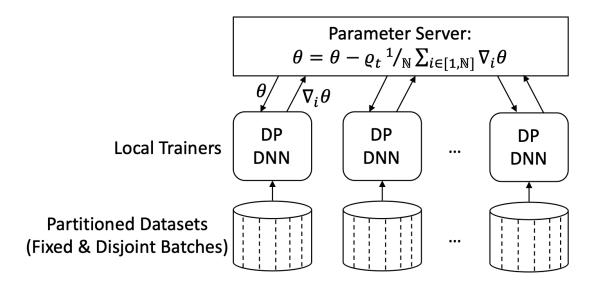
$$\hat{\mathbb{E}}_{lb} f_k(x) > e^{2\epsilon_r} \max_{i:i \neq k} \hat{\mathbb{E}}_{ub} f_i(x) \text{ and}$$

$$\overline{x} = x + Lap(\frac{\kappa \Delta_r^x}{\epsilon_r}), \quad \overline{h} = h + Lap(\frac{\varphi \Delta_r^h}{\epsilon_r})$$

$$\forall \alpha \in l_p(\kappa + \varphi)_{max}: f_k(x + \alpha) > \max_{i:i \neq k} f_i(x + \alpha)$$

where k = y(x), indicating that a small perturbation in the input does not change the predicted label y(x).

### Stochastic Batch Mechanism



Under the same DP protection.

- Training from multiple batches with more adversarial examples, without affecting the DP bound.
- The optimization of one batch does not affect the DP protection at any other batch and at the dataset level D, across T training steps.

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### Experimental Results

- Interplay among model utility, privacy loss, and robustness bounds
  - privacy budget
  - attack sizes
  - scalability

- CNNs on MNIST, CIFAR-10
- ResNet-18 on Tiny ImageNet

- Baseline approaches
  - PixeIDP [Lecuyer et al., S&P'19]
  - DPSGD [Abadi et al., CCS'16]
  - AdLM [Phan et al., ICDM'17]
  - Secure-SGD [Phan et al., IJCAI'19]
     with AGM [Balle et al., ICML'18]

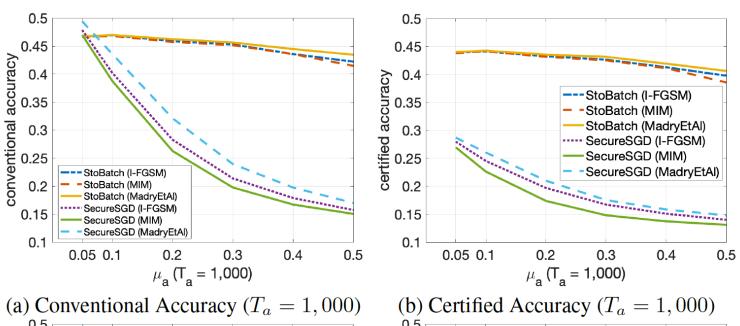
$$conventional\ acc = \sum_{i=1}^{|test|} \frac{isCorrect(x_i)}{|test|}$$

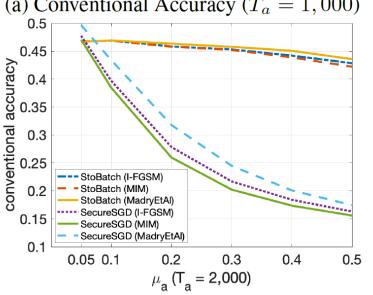
$$\textit{certified acc} = \sum_{i=1}^{|test|} \frac{isCorrect(x_i) \& isRobust(x_i)}{|test|}$$

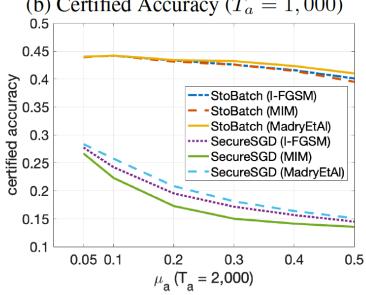
[Lécuyer et al., 2019]

### CIFAR-10

- StoBatch
  - $45.25 \pm 1.6\%$  (conventional)
  - 42.59 ± 1.58% (certified)
- SecureSGD
  - 29.08  $\pm$  11.95% (conventional)
  - 19.58 ± 5.0% (certified)
- p < 2.75e-20
  - 2-tail t-test



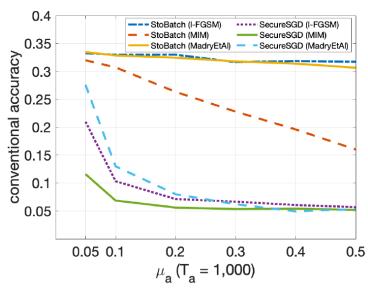


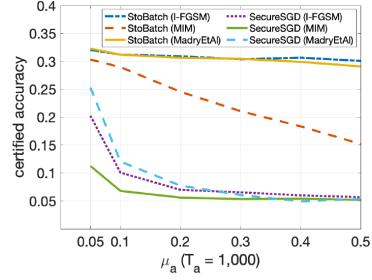


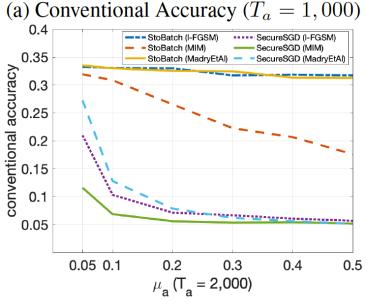
- (c) Conventional Accuracy  $(T_a = 2,000)$
- (d) Certified Accuracy  $(T_a = 2,000)$

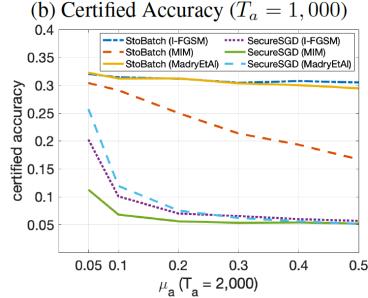
# Tiny ImageNet

- StoBatch
  - 29.78  $\pm$  4.8% (conventional)
  - 28.31 ± 1.58% (certified)
- SecureSGD
  - 8.99 ± 5.95% (conventional)
  - 8.72 ± 5.5% (certified)
- p < 1.55e-42
  - 2-tail t-test









(c) Conventional Accuracy ( $T_a = 2,000$ )

(d) Certified Accuracy ( $T_a = 2,000$ )

Accuracy on the Tiny ImageNet dataset, under Strong Iterative Attacks ( $T_a = 1,000; 2,000$ ).  $\epsilon$  is set to 5.

#### Conclusion

- Established a connection among DP preservation to protect the training data, adversarial learning, and certified robustness.
- Derived a sequential composition robustness in both input and latent spaces.
- Addressed the trade-off among model utility, privacy loss, and robustness.
- Rigorous experiments shown that our mechanism significantly enhances the robustness and scalability of DP DNNs.



# Thank you!

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