

# Learning Similarity Metrics for Numerical Simulations

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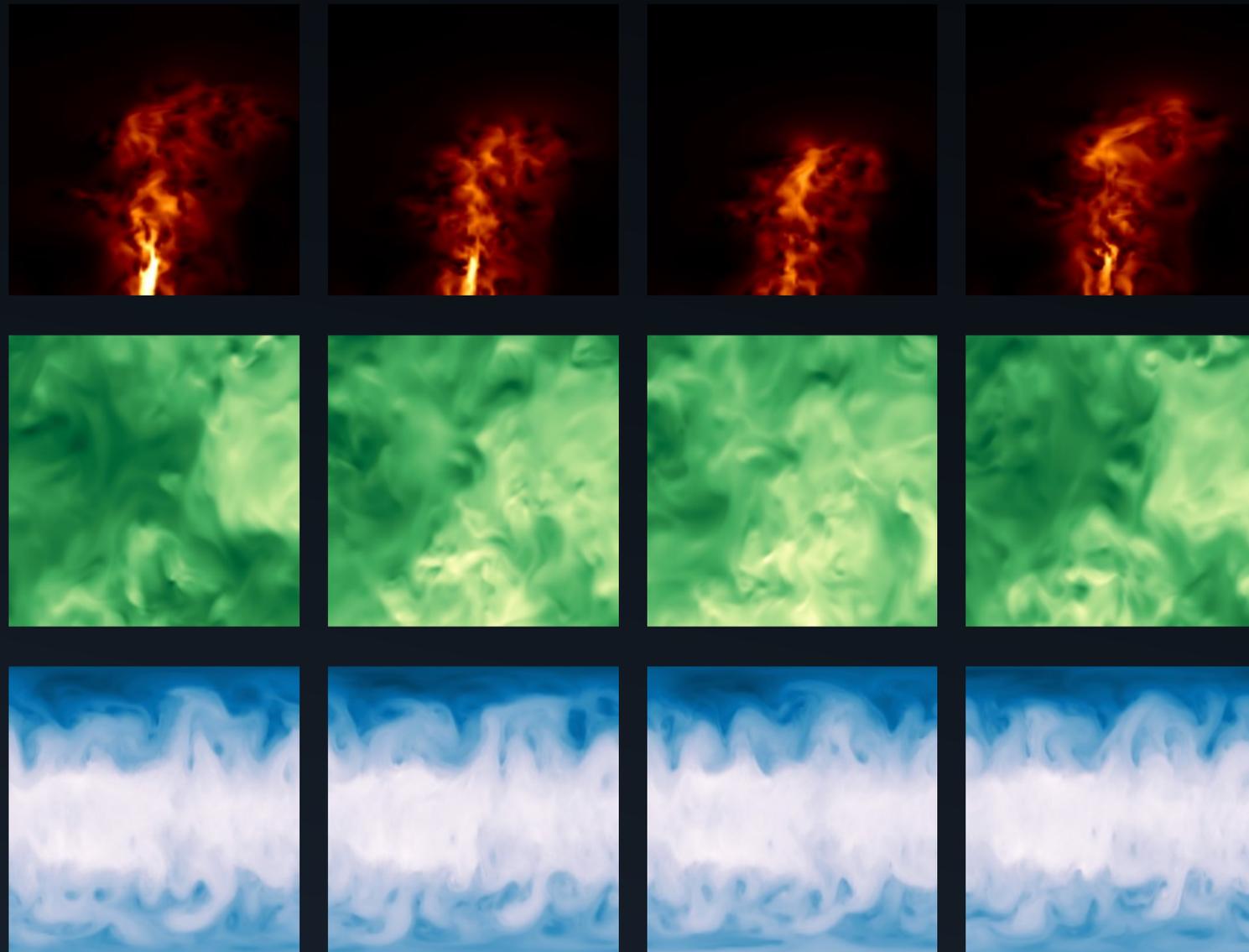
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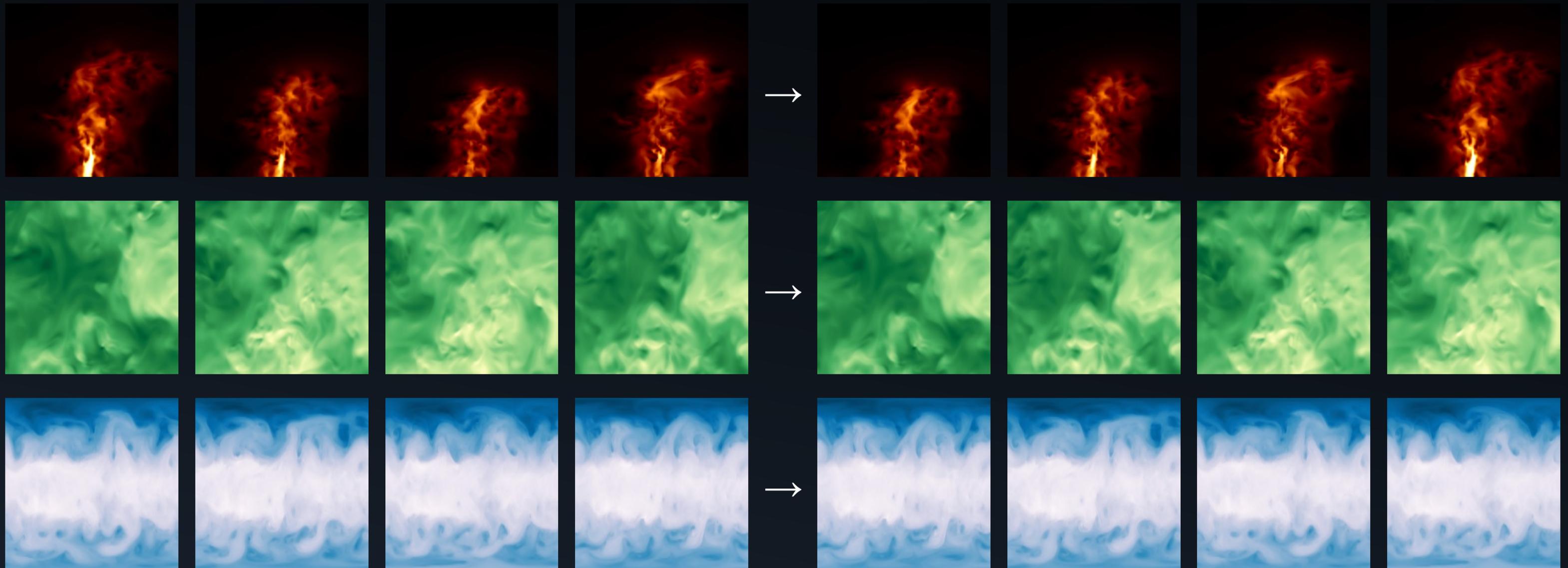
# Overview – Motivation

Similarity assessment of scalar 2D simulation data from PDEs



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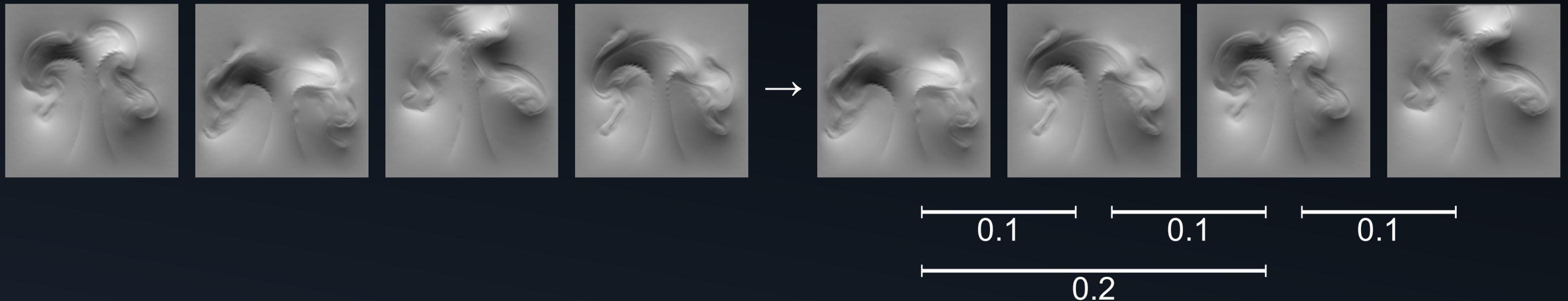


# Overview – Motivation

Typical metrics like  $L^1$  or  $L^2$  operate locally  $\rightarrow$  structures and patterns are ignored

Recognition of spatial contexts with CNNs

Mathematical metric properties should be considered



# Overview – Method

## Numerical Simulation

$[p_i \quad p_{i+1\Delta_i} \quad p_{i+2\Delta_i} \quad \dots \quad p_{i+10\Delta_i}]$

Initial condition varied in isolation

Variation defines

Simulation with PDE Solver



Data sequence

$[0.0 \quad 0.1 \quad 0.2 \quad \dots \quad 1.0]$

Ground truth distances

# Overview – Method

## Numerical Simulation

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Simulation with PDE Solver

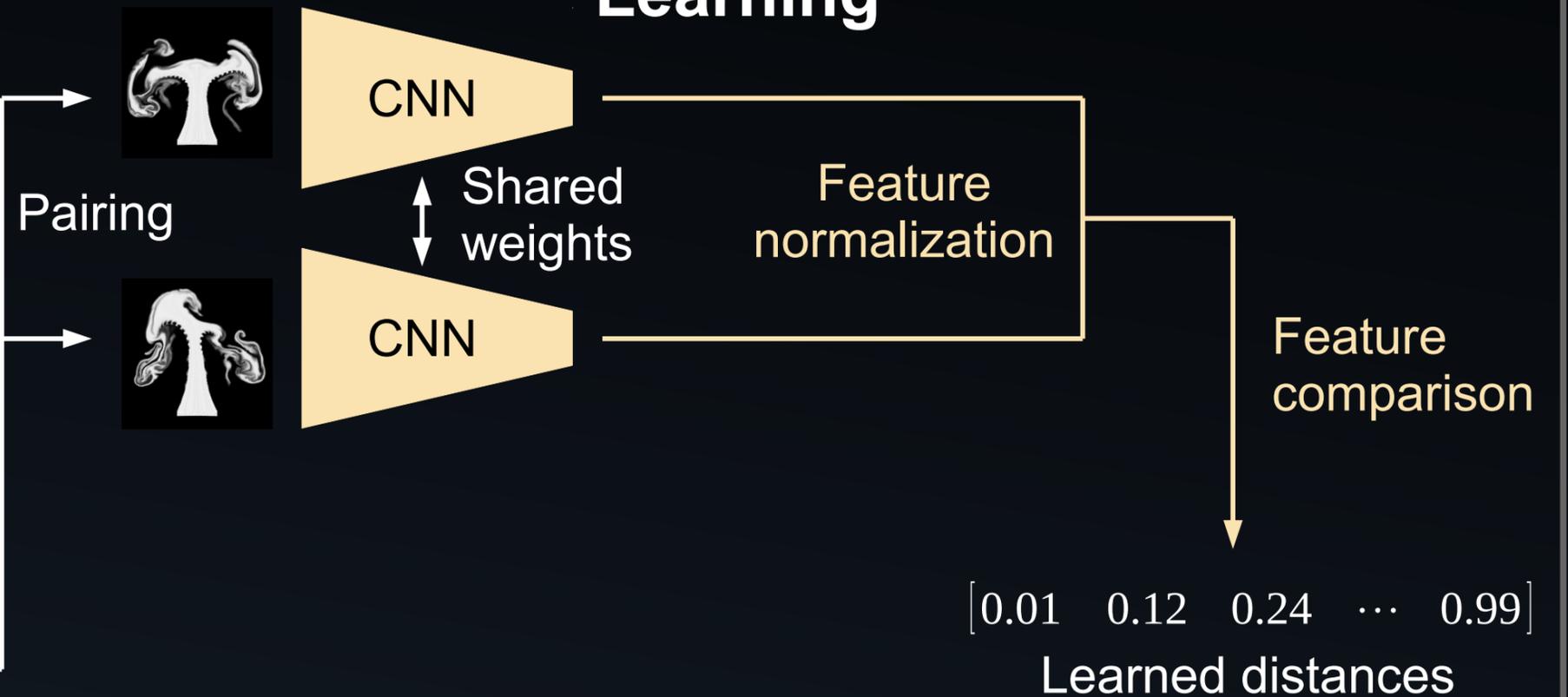


Data sequence

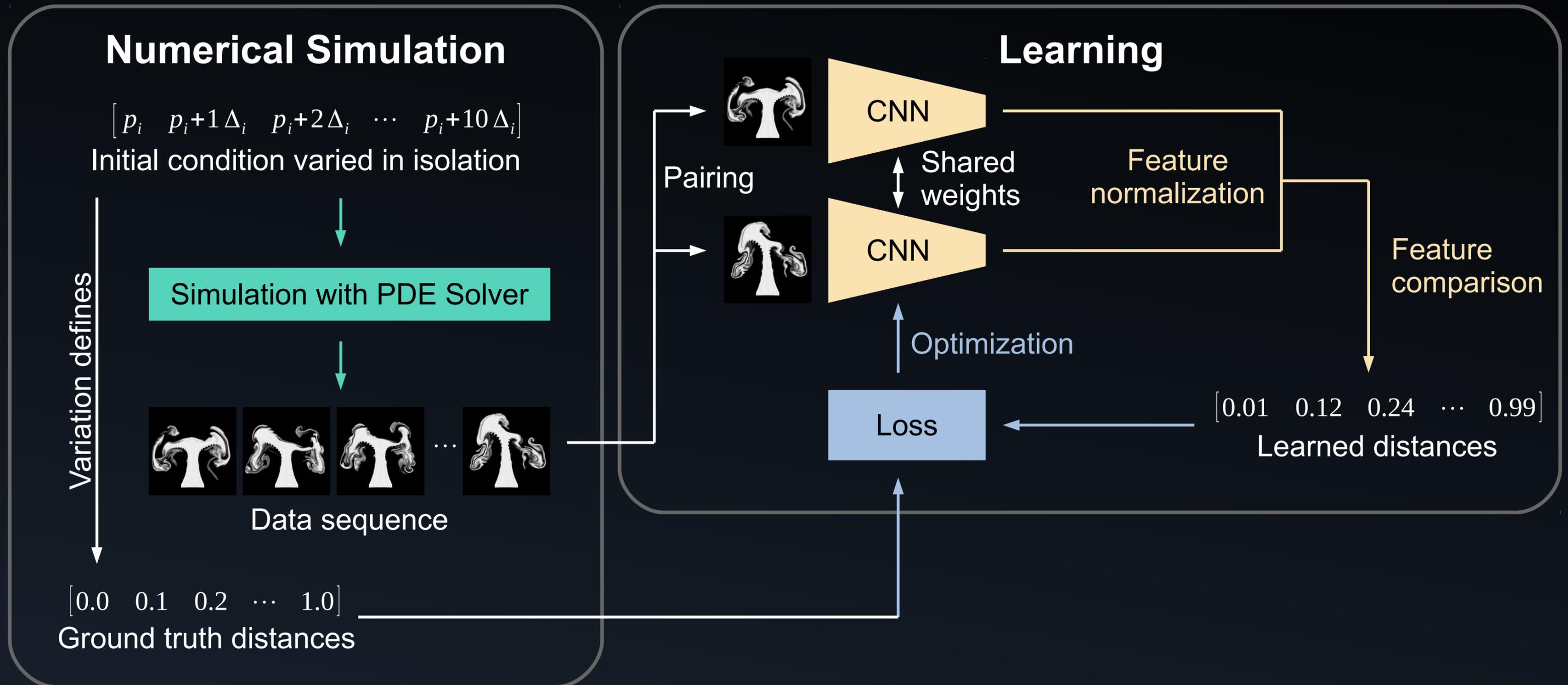
$[0.0 \quad 0.1 \quad 0.2 \quad \dots \quad 1.0]$   
Ground truth distances

Variation defines

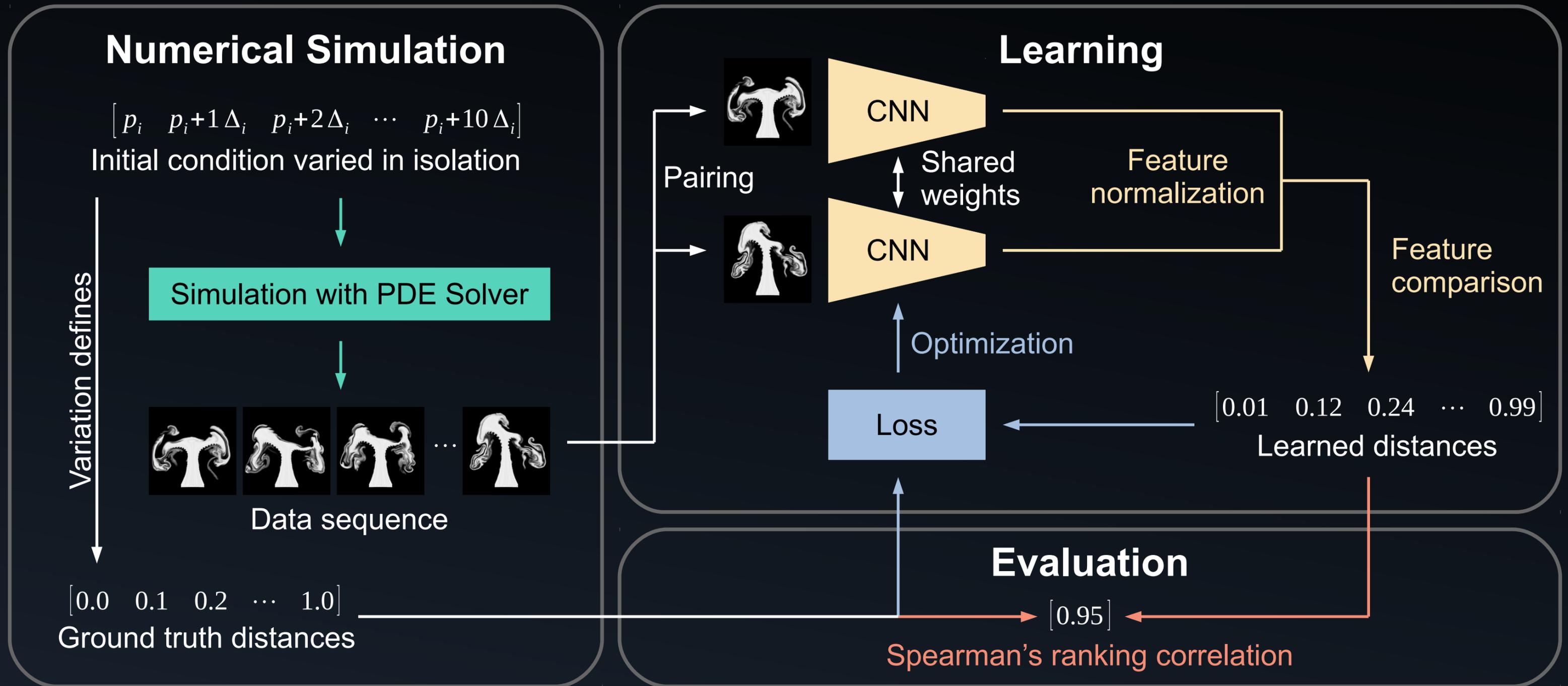
## Learning



# Overview – Method



# Overview – Method



# Overview – Results

Single example: distance comparison

Plume (a)



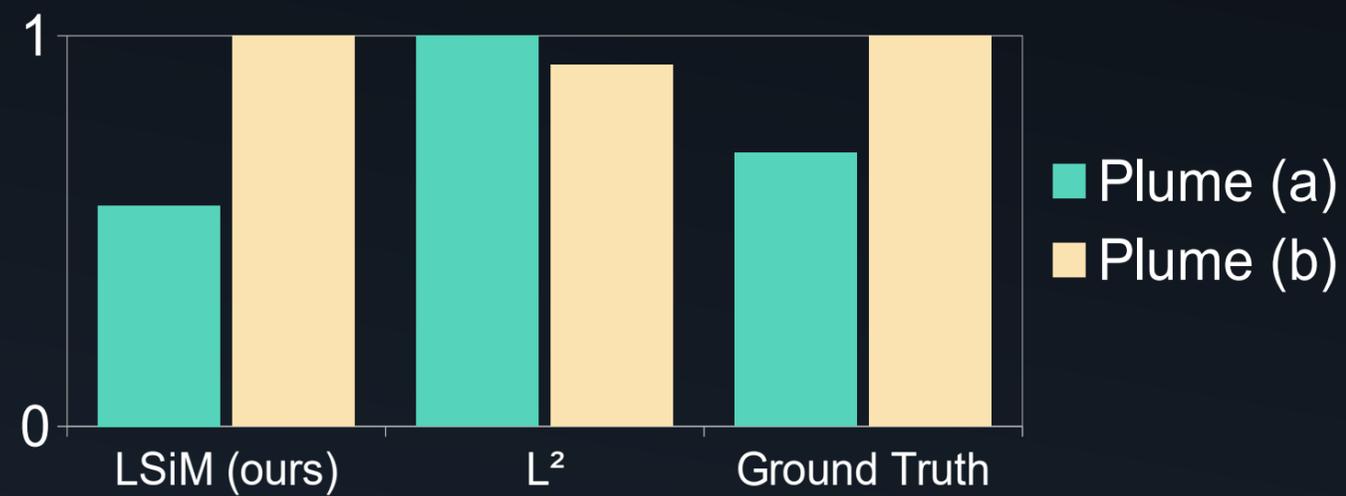
Reference



Plume (b)

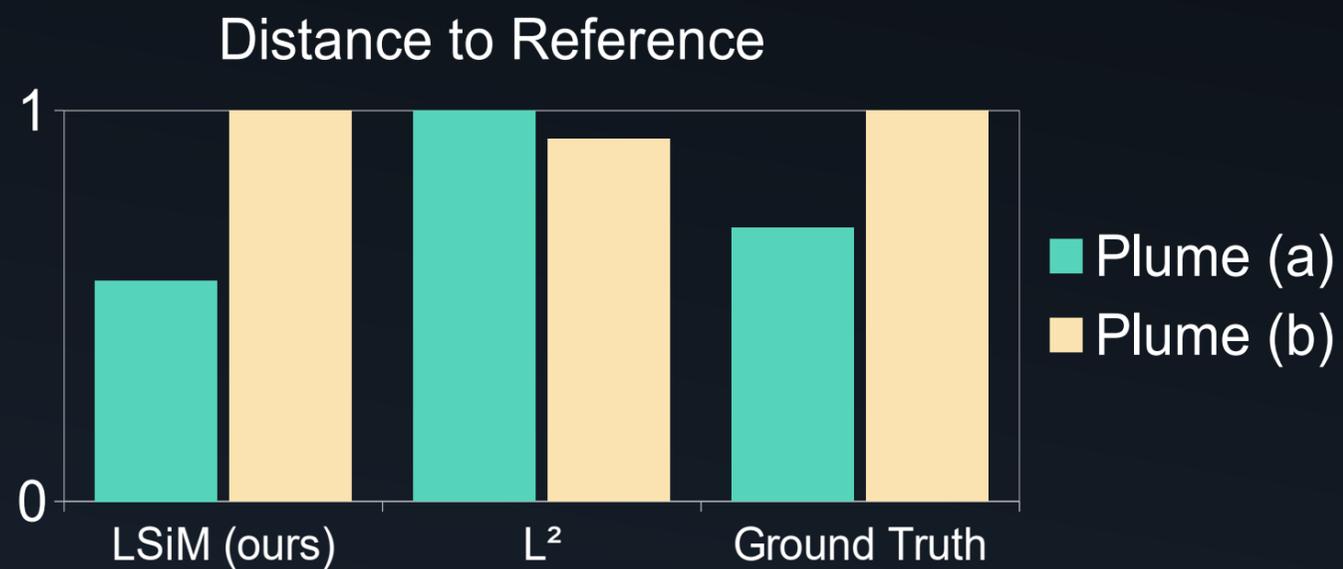
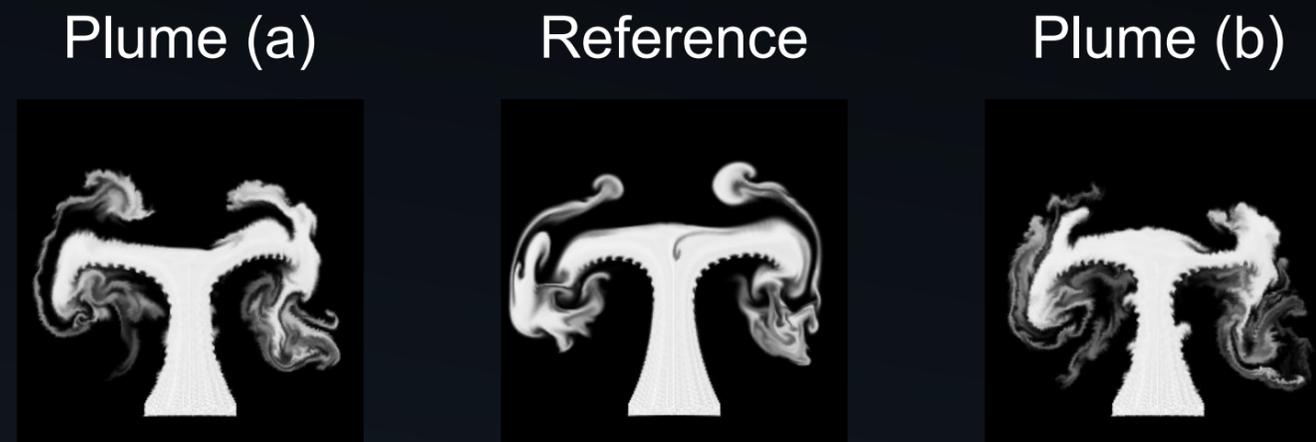


Distance to Reference



# Overview – Results

Single example: distance comparison



Combined test data: correlation evaluation



# Related Work

## “Shallow” vector space metrics

- Metrics induced by  $L^p$ -norms, peak signal-to-noise ratio (PSNR)
- Structural similarity index (SSIM) [Wang04]

## Evaluation with user studies for PDE data

- Liquid simulations [Um17]
- Non-oscillatory discretization schemes [Um19]

## Image-based deep metrics with CNNs

- E.g. learned perceptual image patch similarity (LPIPS) [Zhang18]

[Wang04] Wang, Bovik, Sheikh, and Simoncelli. Image quality assessment: From error visibility to structural similarity. *IEEE Transactions on Image Processing*, 2004

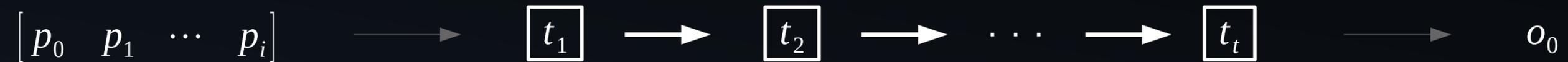
[Um17] Um, Hu, and Thuerey. Perceptual Evaluation of Liquid Simulation Methods. *ACM Transactions on Graphics*, 2017

[Um19] Um, Hu, Wang, and Thuerey. Spot the Difference: Accuracy of Numerical Simulations via the Human Visual System. *CoRR*, abs/1907.04179, 2019

[Zhang18] Zhang, Isola, Efros, Shechtman, and Wang. The Unreasonable Effectiveness of Deep Features as a Perceptual Metric. *CVPR*, 2018

# Data Generation

Time depended, motion-based PDE with one varied initial condition



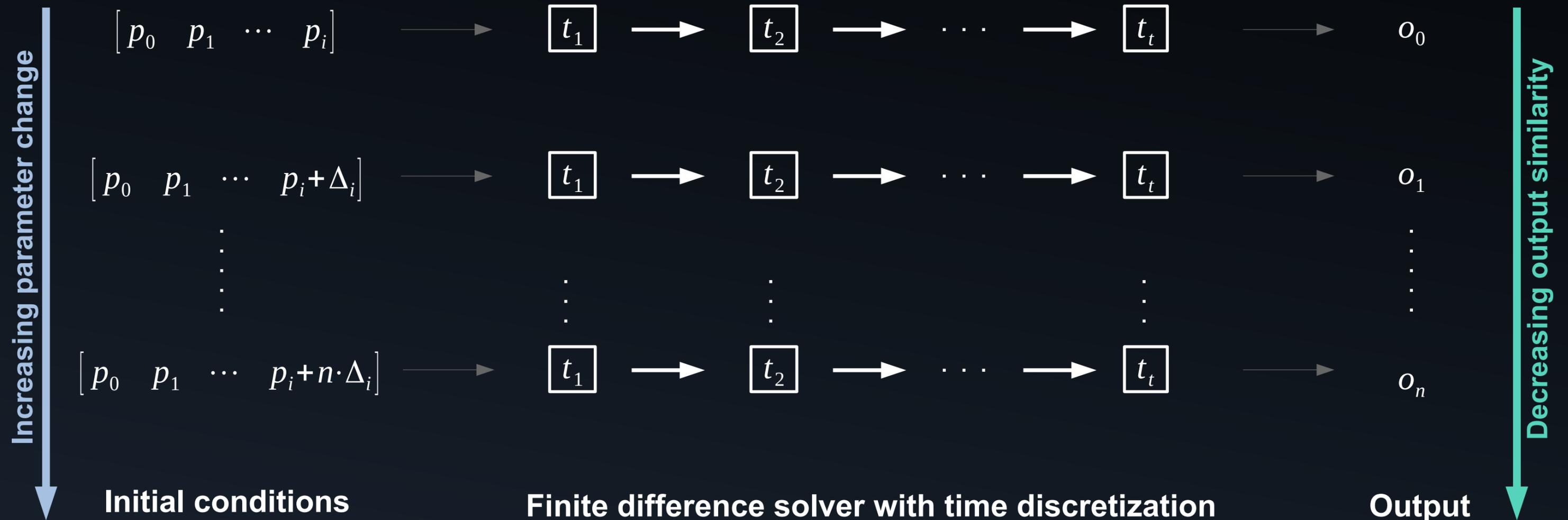
Initial conditions

Finite difference solver with time discretization

Output

# Data Generation

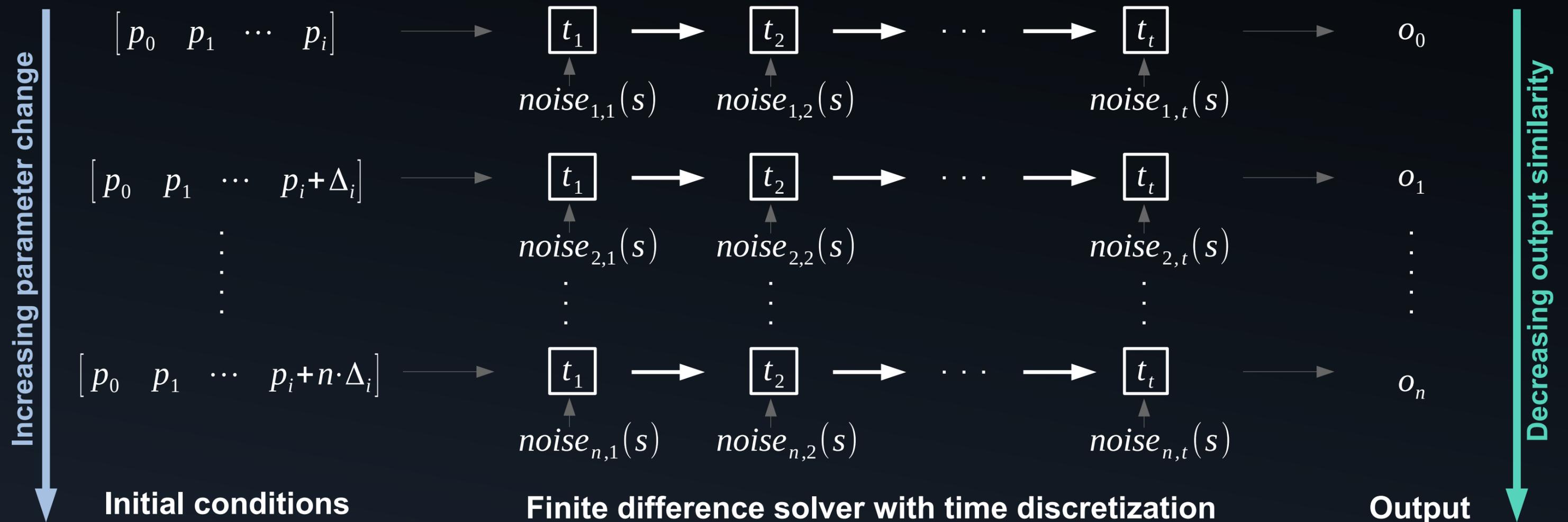
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# Data Generation

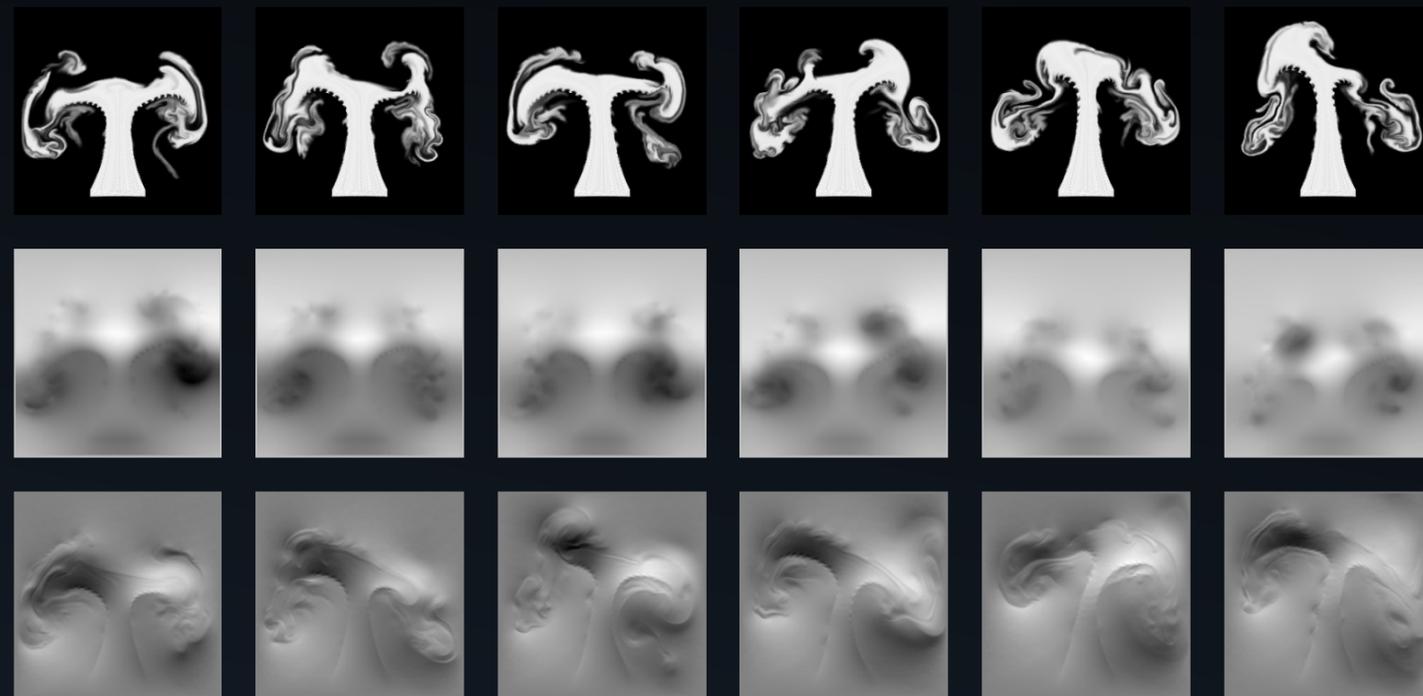
Time depended, motion-based PDE with one varied initial condition

Chaotic behavior in controlled environment → added noise to adjust data difficulty

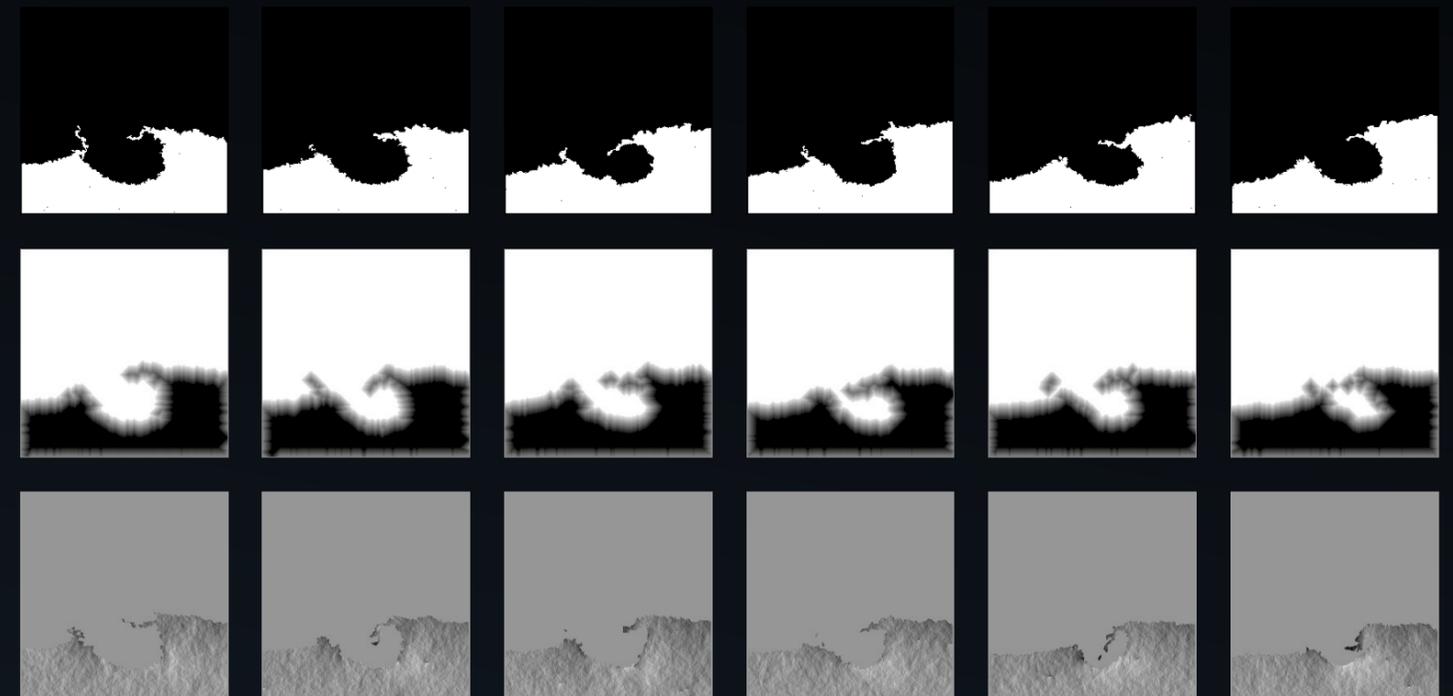


# Training Data

## Eulerian smoke plume



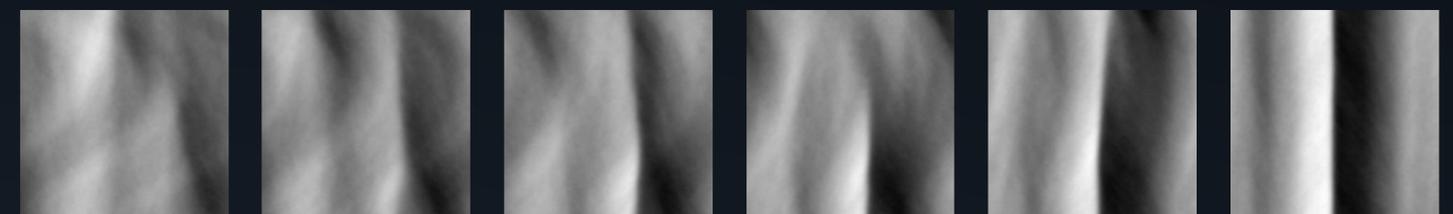
## Liquid via FLIP [Zhu05]



## Advection-diffusion transport

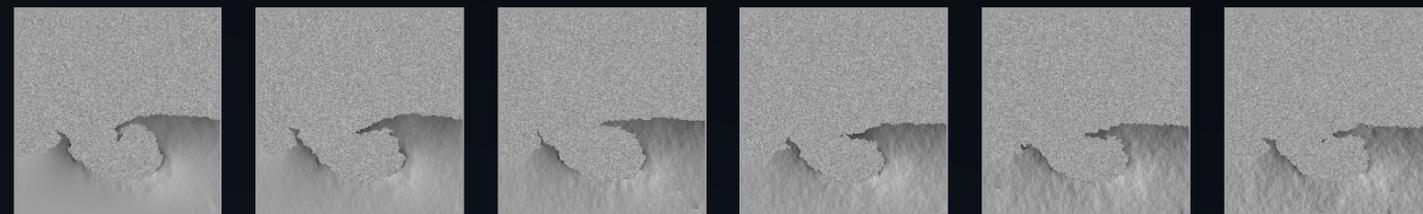


## Burger's equation

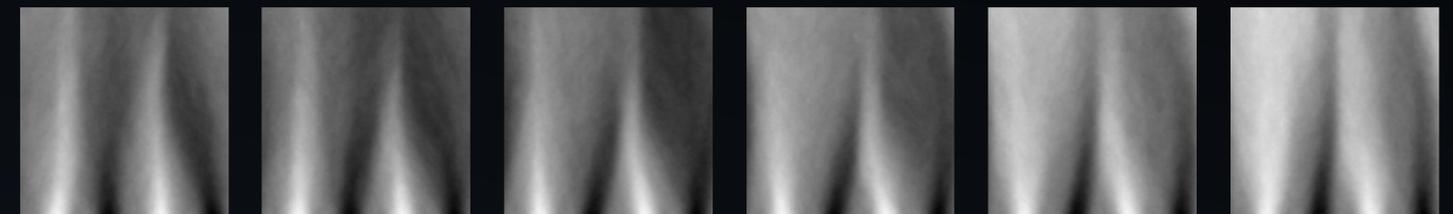


# Test Data

Liquid (background noise)



Advection-diffusion transport (density)



Shape data



Video data



TID2013 [Ponomarenko15]

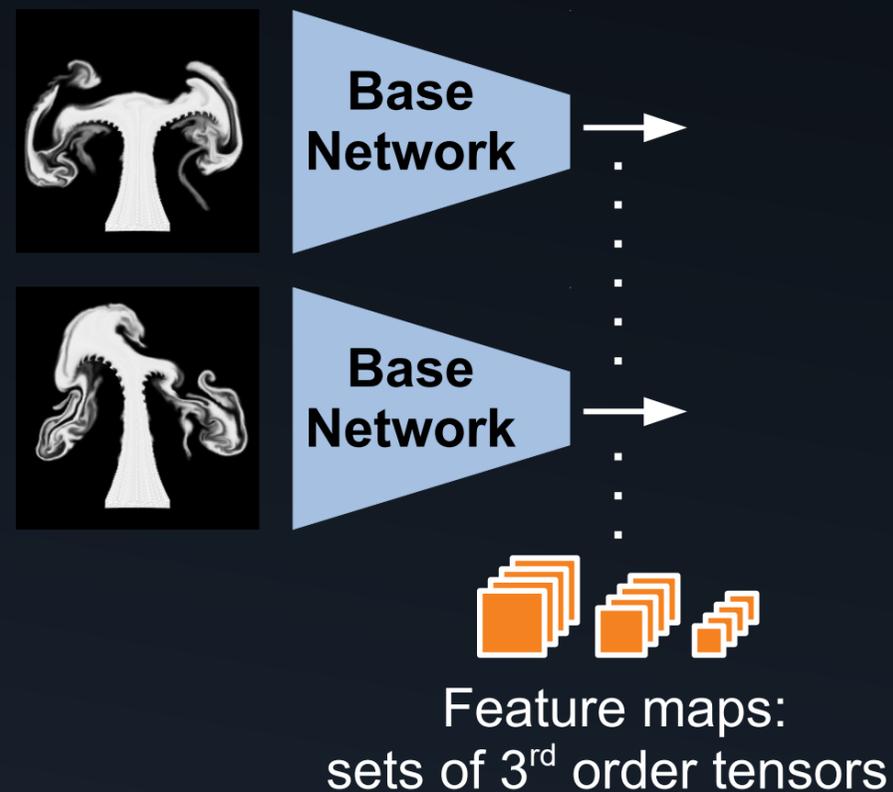


# Method – Base Network

Siamese architecture (shared weights) → Convolution + ReLU layers

Feature extraction from both inputs

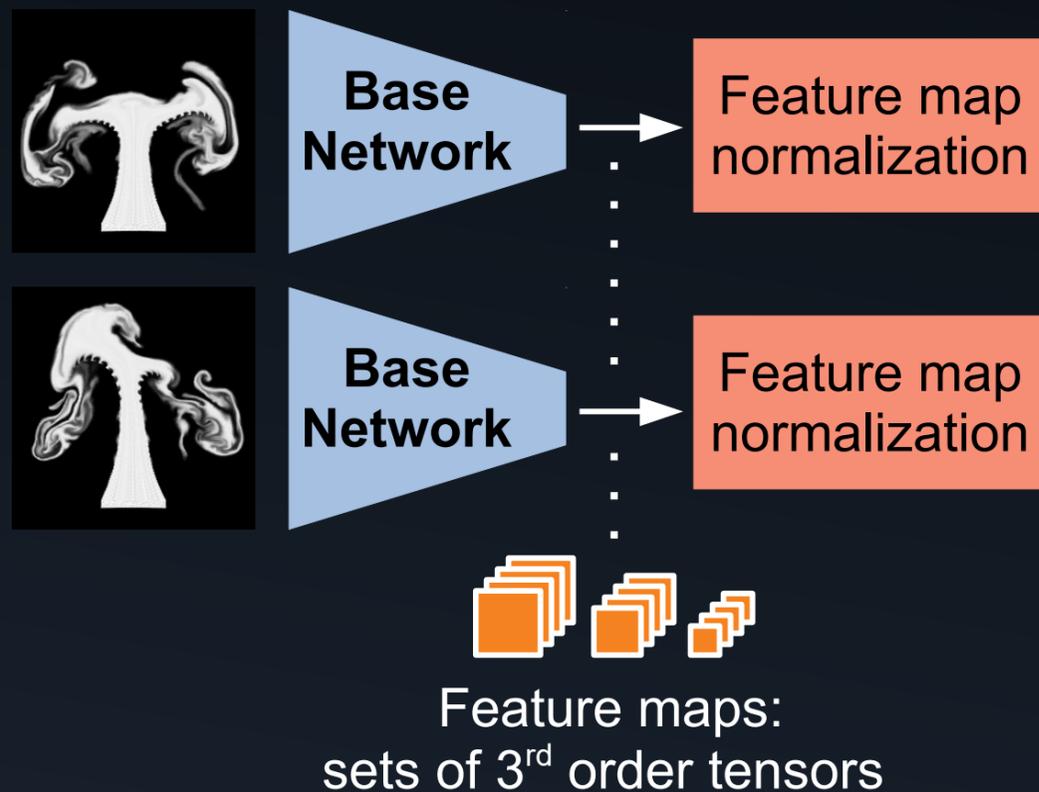
Existing network possible → specialized model works better



# Method – Feature Normalization

Adjust value range of feature vectors along channel dimension

- Unit length normalization → cosine distance (only angle comparison)
- Element-wise std. normal distribution → angle and length in global length distribution

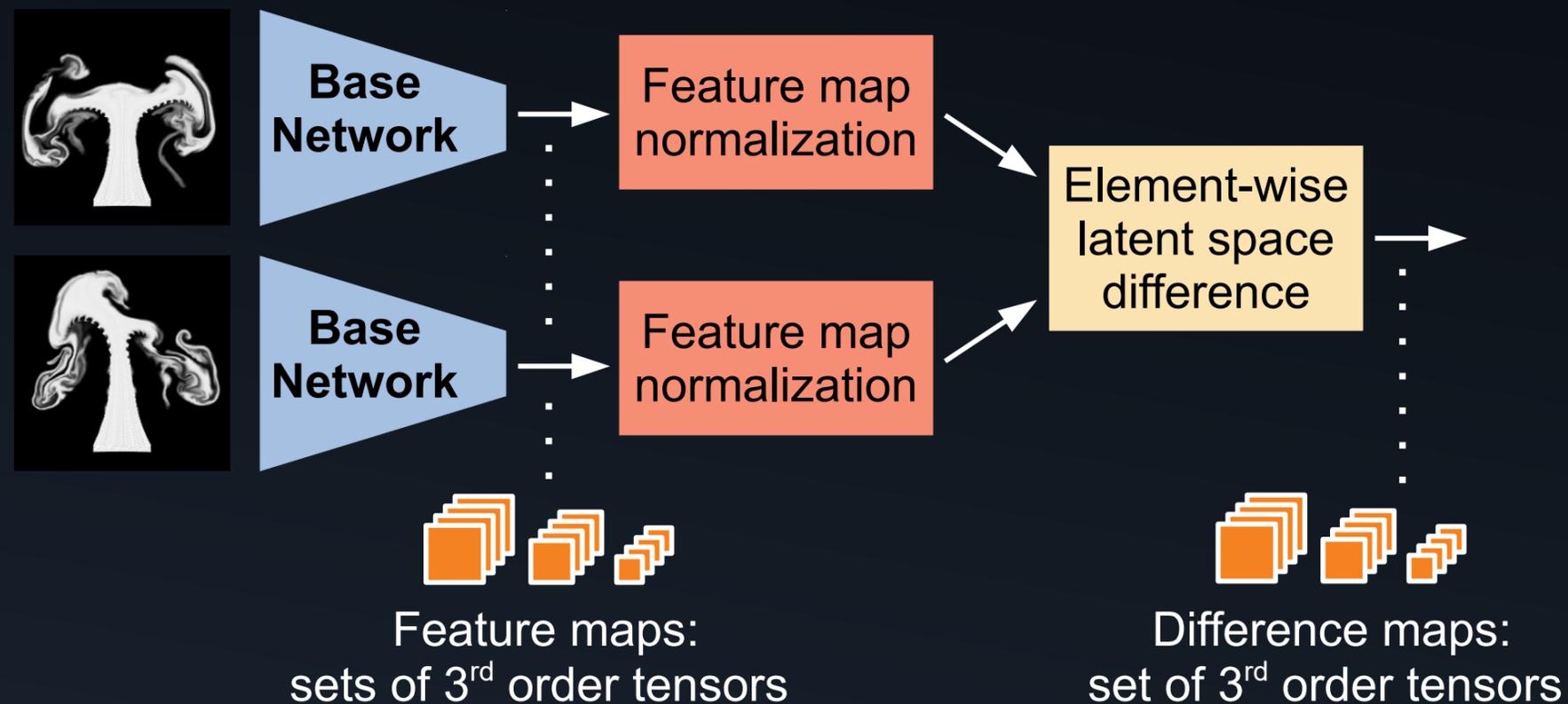


# Method – Latent Space Difference

Actual comparison of feature maps → element-wise distance

Must be a metric w.r.t. the latent space → ensure metric properties

$|\tilde{x} - \tilde{y}|$  or  $(\tilde{x} - \tilde{y})^2$  are useful options

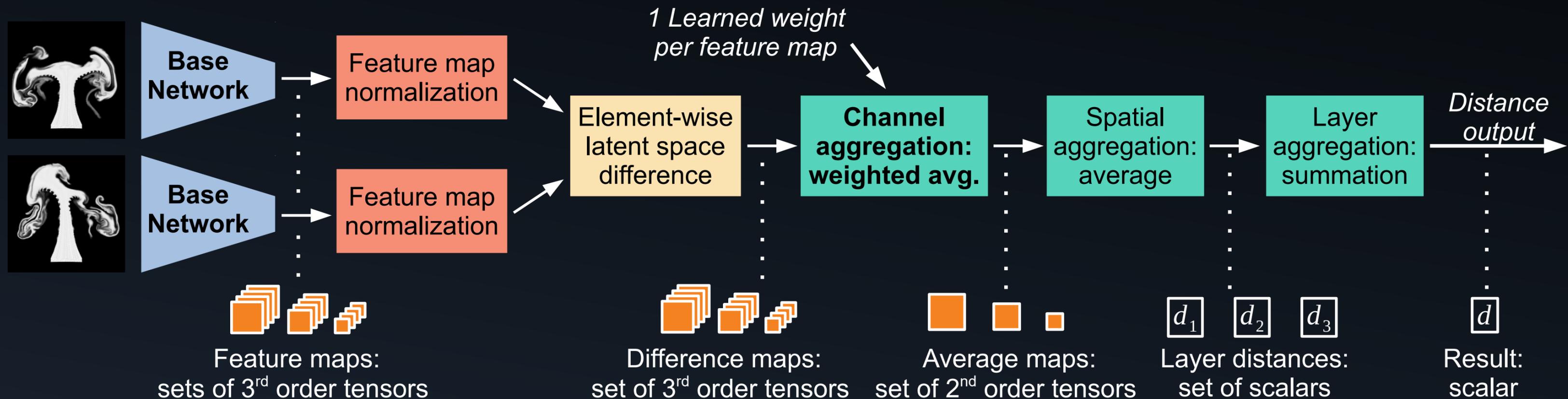


# Method – Aggregations

Compression of difference maps to scalar distance prediction

Learned channel aggregation via weighted average

Simple aggregations with sum or average



# Loss Function

Ground truth distances  $c$  and predicted distances  $d$

$$L(c, d) = \lambda_1 (c - d)^2 + \lambda_2 \left( 1 - \frac{(c - \bar{c}) \cdot (d - \bar{d})}{\|c - \bar{c}\|_2 \|d - \bar{d}\|_2} \right)$$

Mean squared error term  $\rightarrow$  minimize distance deviation directly

Inverted correlation term  $\rightarrow$  maximize linear distance relationship

# Results

Evaluation with Spearman's rank correlation

Ground truth against predicted distances

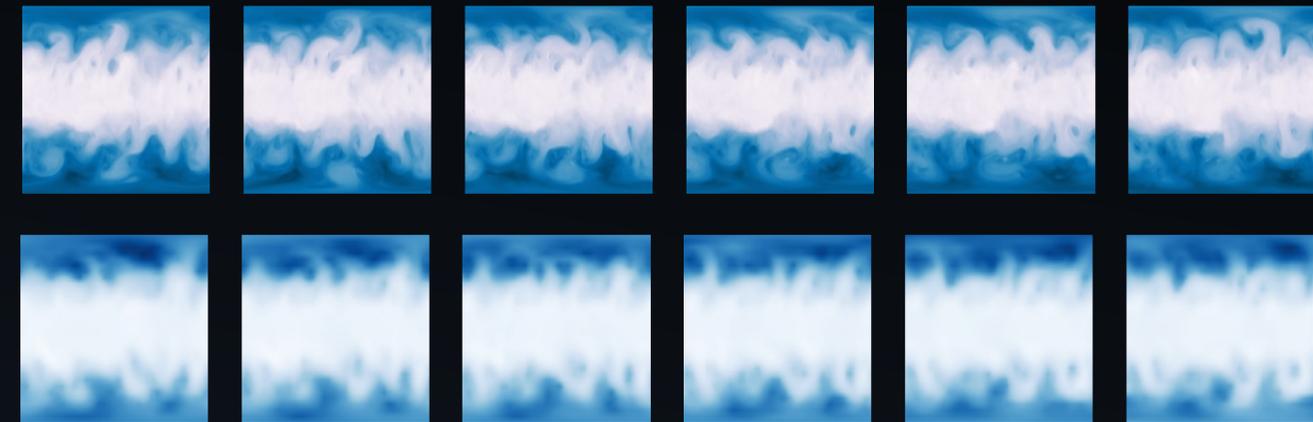
Metric	Validation data sets				Test data sets					
	<i>Smo</i>	<i>Liq</i>	<i>Adv</i>	<i>Bur</i>	<i>TID</i>	<i>LiqN</i>	<i>AdvD</i>	<i>Sha</i>	<i>Vid</i>	<i>All</i>
$L^2$	0.66	0.80	0.74	0.62	0.82	0.73	0.57	0.58	0.79	0.61
<i>SSIM</i>	0.69	0.74	0.77	0.71	0.77	0.26	0.69	0.46	0.75	0.53
<i>LPIPS</i>	0.63	0.68	0.68	0.72	0.86	0.50	0.62	0.84	0.83	0.66
<i>LSiM</i>	0.78	0.82	0.79	0.75	0.86	0.79	0.58	0.88	0.81	0.73

# Real-world Evaluation

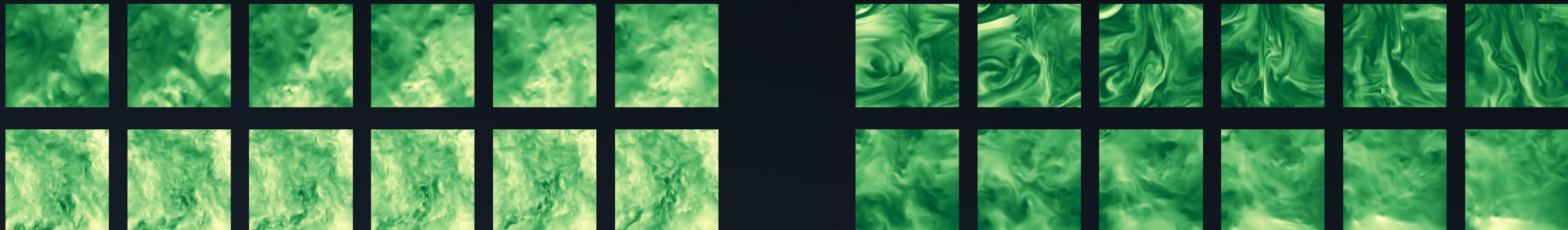
ScalarFlow [Eckert19]



WeatherBench [Rasp20]



Johns Hopkins Turbulence Database (JHTDB) [Perlman07]



[Eckert19] Eckert, Um, and Thuerey. Scalarflow: A large-scale volumetric data set of real-world scalar transport flows [...]. *ACM Transactions on Graphics*, 2019  
 [Rasp20] Rasp, Dueben, Scher, Weyn, Mouatadid, and Thuerey. Weatherbench: A benchmark dataset for data-driven weather forecasting. *CoRR*, abs/2002.00469, 2020  
 [Perlman07] Perlman, Burns, Li, and Meneveau. Data exploration of turbulence simulations using a database cluster. *ACM/IEEE Conference on Supercomputing*, 2007

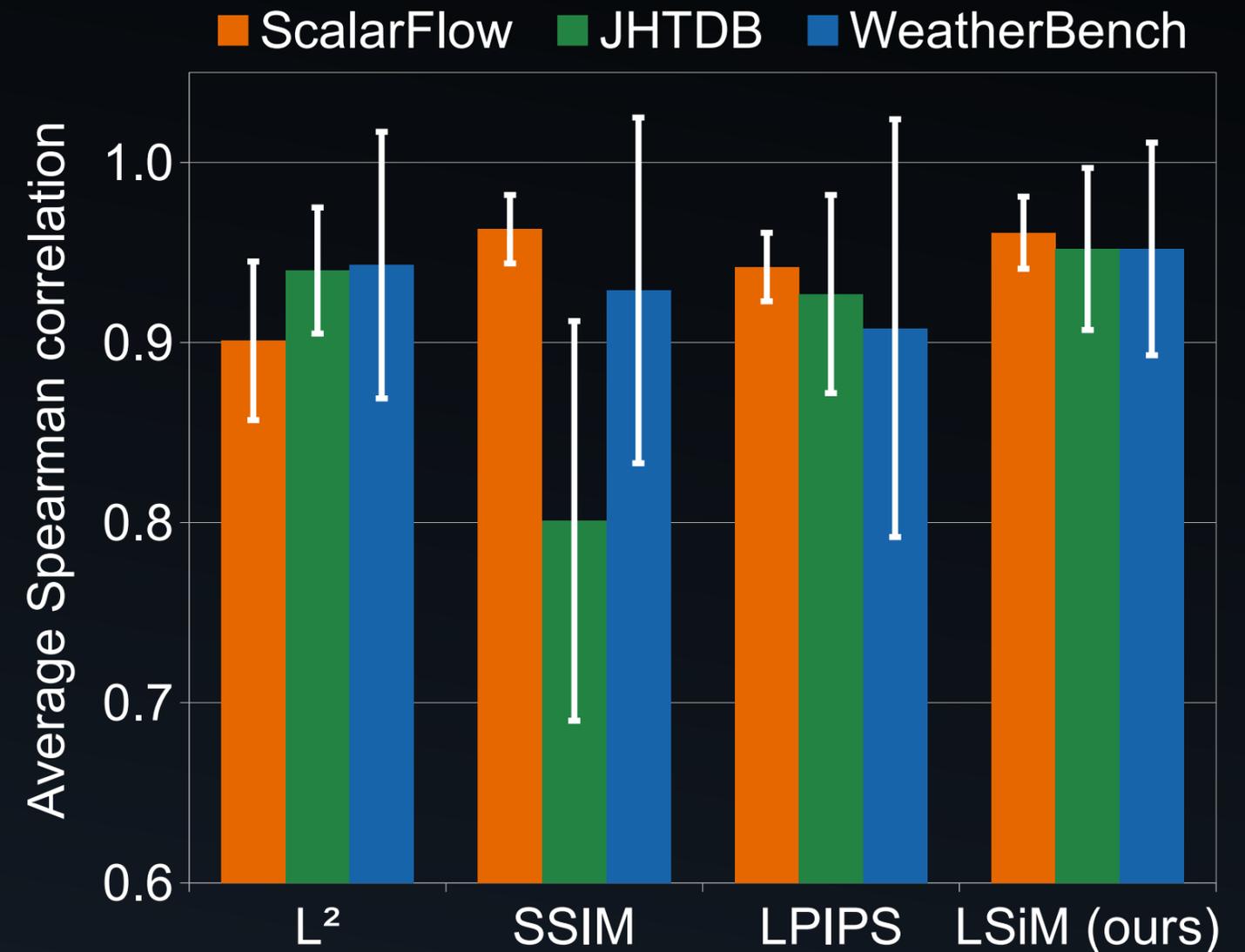
# Real-world Evaluation

Retrieve order of spatial and temporal frame translations

Six interval spacings per data repository

180-240 sequences each

Mean and standard deviation over correlation of each spacing



# Future Work

Accuracy assessment of new simulation methods

Parameter reconstructions of observed behavior

Guiding generative models of physical systems

Extensions to other data

- 3D flows and further PDEs
- Multi-channel turbulence data

# Thank you for your attention!

Join the live-sessions for questions and discussion

Source code available at <https://github.com/tum-pbs/LSIM>

