

# Soft Threshold Weight Reparameterization for Learnable Sparsity

**Aditya Kusupati**

Vivek Ramanujan\*, Raghav Somani\*, Mitchell Wortsman\*

Prateek Jain, Sham Kakade and Ali Farhadi

# Motivation

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- Deep Neural Networks
  - Highly accurate
  - Millions of parameters & Billions of FLOPs
  - Expensive to deploy
- Sparsity
  - Reduces model size & inference cost
  - Maintains accuracy
  - Deployment on CPUs & weak single-core devices

Privacy preserving  
smart glasses



Billions of mobile  
devices



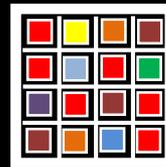
# Motivation

- Existing sparsification methods
  - Focus on model size vs accuracy – *very little on inference FLOPs*
  - Global, uniform or heuristic sparsity budget across layers

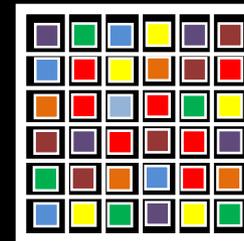
Layer 1



Layer 2



Layer 3



	Layer 1	Layer 2	Layer 3	Total
# Params	20	100	1000	1120
FLOPs	100K	100K	50K	250K

## Sparsity – Method 1

# Params	20	100	100	220
FLOPs	100K	100K	5K	205K

## Sparsity – Method 2

# Params	10	10	200	220
FLOPs	50K	10K	10K	70K

# Motivation

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- Non-uniform sparsity budget – *Layer-wise*
  - Very hard to search in deep networks
  - Sweet spot – *Accuracy vs FLOPs vs Sparsity*
  - Existing techniques
    - Heuristics – *increase FLOPs*
    - Use RL – *expensive to train*

*“Can we design a robust efficient method to learn non-uniform sparsity budget across layers?”*

# Overview

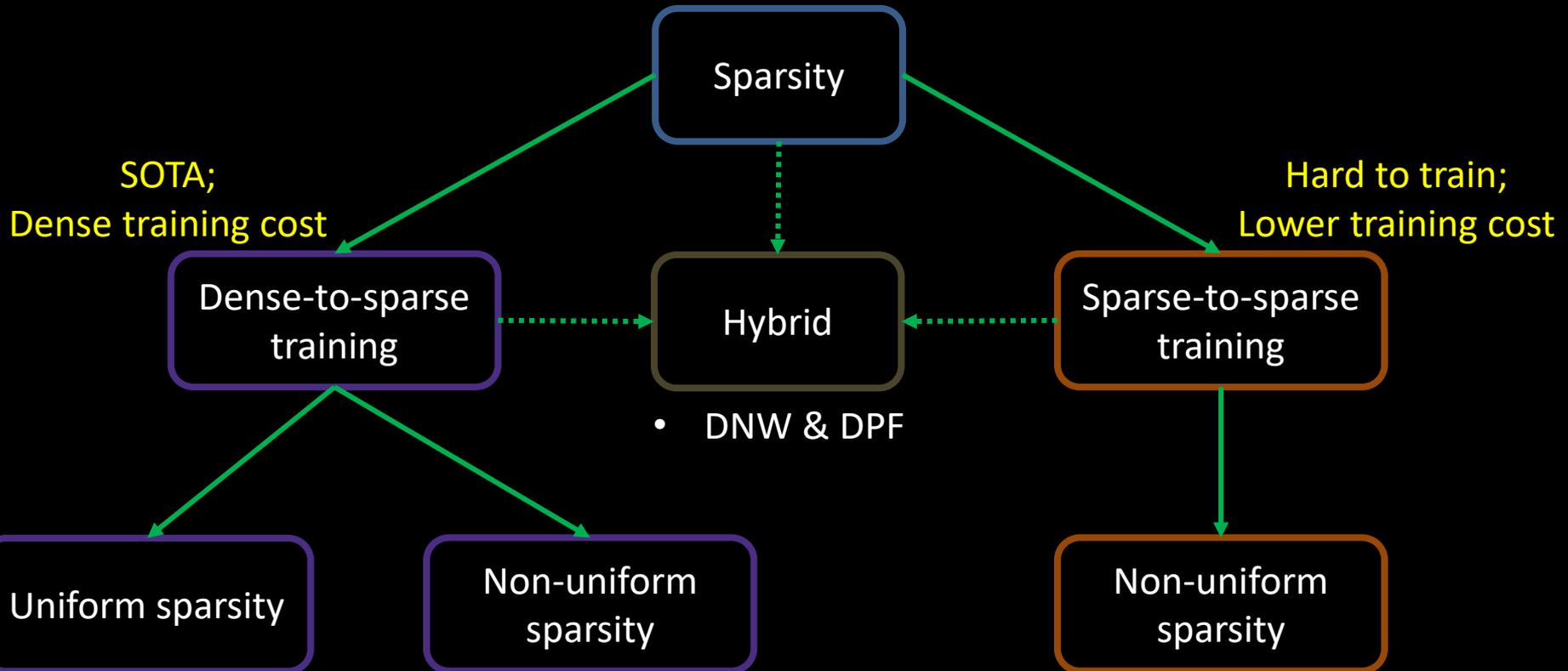
- **STR – Soft Threshold Reparameterization**

$$STR(\mathbf{W}_l, \alpha_l) = \text{sign}(\mathbf{W}_l) \cdot \text{ReLU}(|\mathbf{W}_l| - \alpha_l)$$



- **Learns layer-wise non-uniform sparsity budgets**
  - Same model size; Better accuracy; Lower inference FLOPs
  - SOTA on ResNet50 & MobileNetV1 for ImageNet-1K
  - Boosts accuracy by up to 10% in ultra-sparse (98-99%) regime
- **Extensions to structured, global & per-weight (mask-learning) sparsity**

# Existing Methods



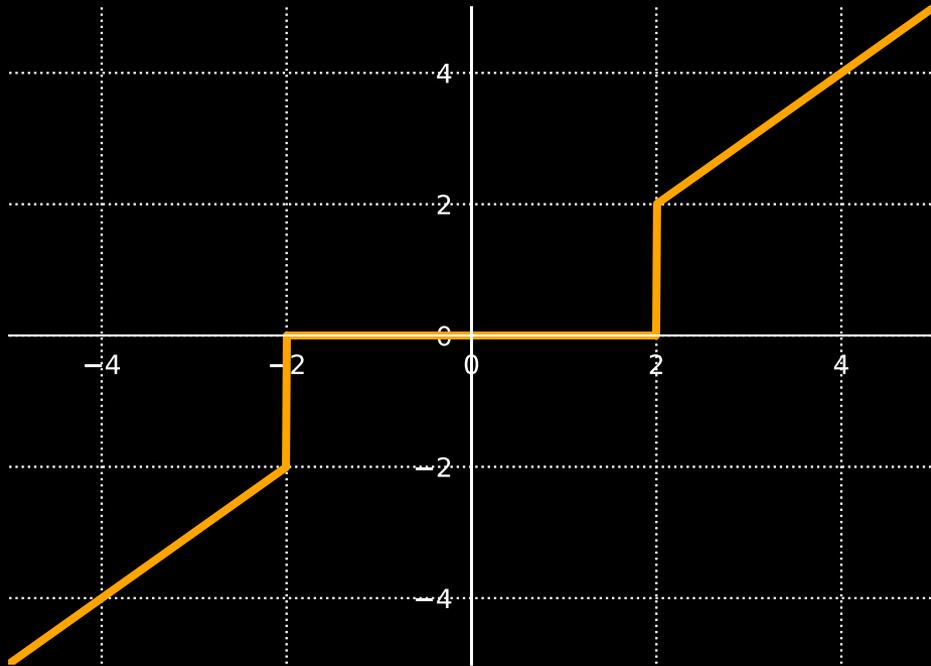
- Gradual Magnitude Pruning (GMP)

- Heuristics – *ERK*
- Global Pruning/Sparsity
- **STR** - some gains from sparse-to-sparse

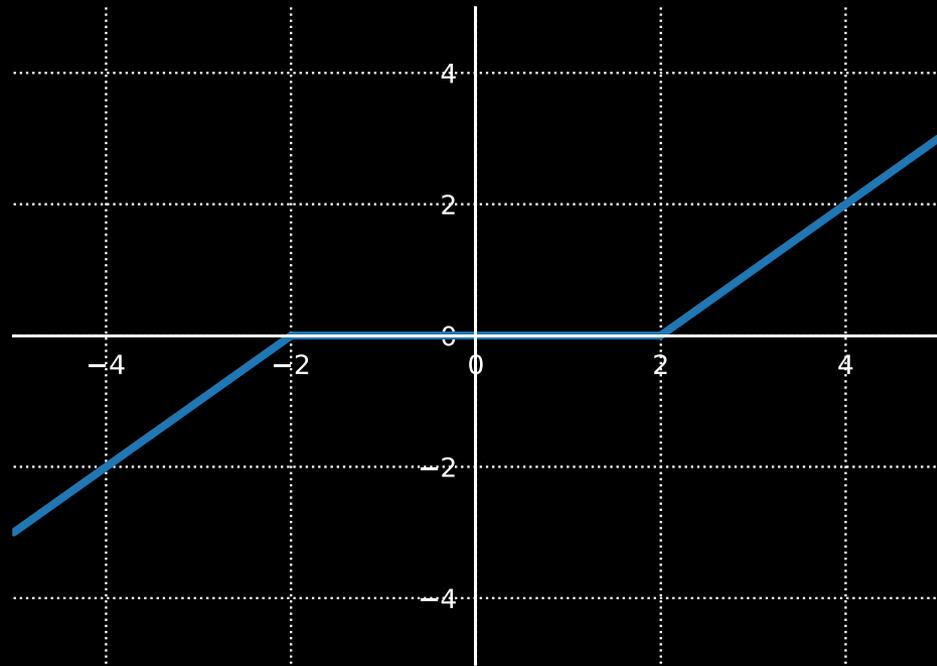
- DSR, SNFS, RigL etc.,
- Heuristics – *ERK*
- Re-allocation using magnitude/gradient

# STR - Method

Hard threshold



Soft threshold



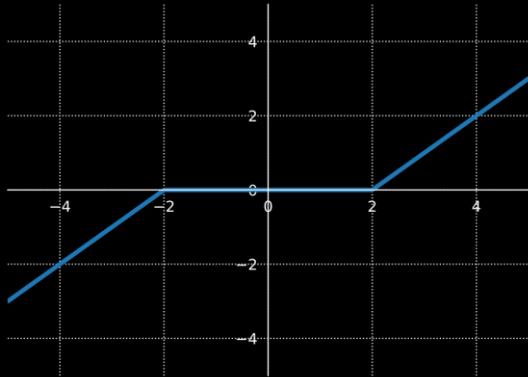
$\alpha = 2$

$$HT(x, \alpha) = \begin{cases} x; & |x| > \alpha \\ 0; & |x| \leq \alpha \end{cases}$$

$$ST(x, \alpha) = \begin{cases} x - \alpha; & x > \alpha \\ 0; & |x| \leq \alpha \\ x + \alpha; & x < -\alpha \end{cases}$$

# STR - Method

Soft threshold



$$\begin{aligned} ST(x, \alpha) &= \text{sign}(x) \cdot \text{ReLU}(|x| - \alpha) \\ &= \text{sign}(x) \cdot \text{ReLU}(|x| - g(s)) \end{aligned}$$

$L$ -layer DNN,  $\mathcal{W} = [\mathbf{W}_l]_{l=1}^L$ ,  $\mathbf{s} = [s_l]_{l=1}^L$  and a function  $g(\cdot)$

$$\mathcal{S}_g(\mathbf{W}_l, s_l) = \text{sign}(\mathbf{W}_l) \cdot \text{ReLU}(|\mathbf{W}_l| - g(s_l))$$

$$\mathcal{W} \leftarrow \mathcal{S}_g(\mathcal{W}, \mathbf{s})$$

# STR - Training

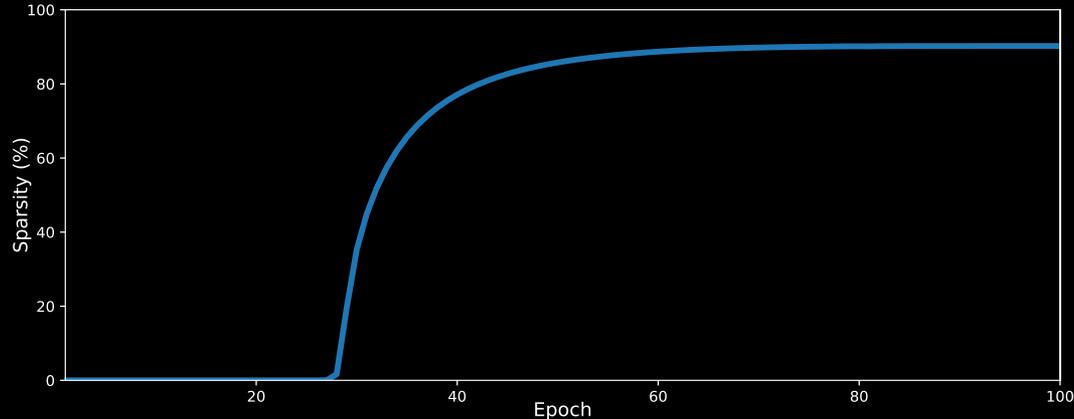
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$$\min_{\mathcal{W}, \mathbf{s}} \mathcal{L}(\mathcal{S}_g(\mathcal{W}, \mathbf{s}), \mathcal{D}) + \lambda \sum_{l=1}^L (|\mathbf{W}_l|_2^2 + |s_l|_2^2)$$

- Regular training with reparameterized weights  $\mathcal{S}_g(\mathcal{W}, \mathbf{s})$
- Same weight-decay parameter ( $\lambda$ ) for both  $(\mathcal{W}, \mathbf{s})$ 
  - Controls the overall sparsity
- Initialize  $s$ ;  $g(s) \approx 0$ 
  - Finer sparsity and dense training control
- Choice of  $g(\cdot)$ 
  - *Unstructured sparsity*: Sigmoid
  - *Structured sparsity*: Exponential

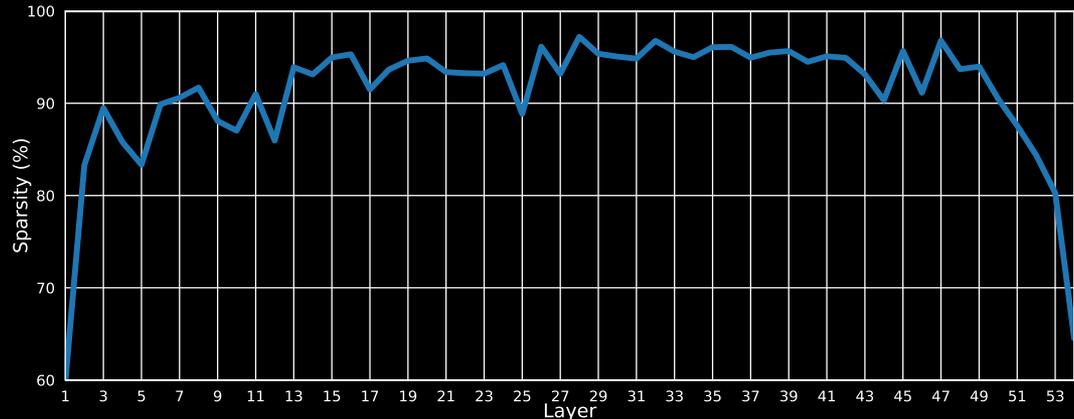
# STR - Training

- STR learns the SOTA hand-crafted heuristic of GMP



Overall sparsity vs Epochs – 90% sparse ResNet50 on ImageNet-1K

- STR learns diverse non-uniform layer-wise sparsities



Layer-wise sparsity – 90% sparse ResNet50 on ImageNet-1K

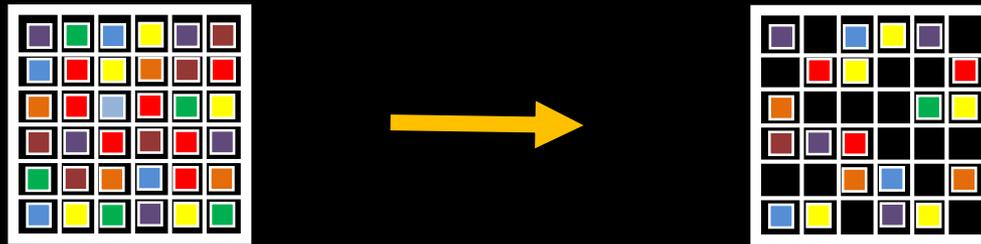
# STR - Experiments

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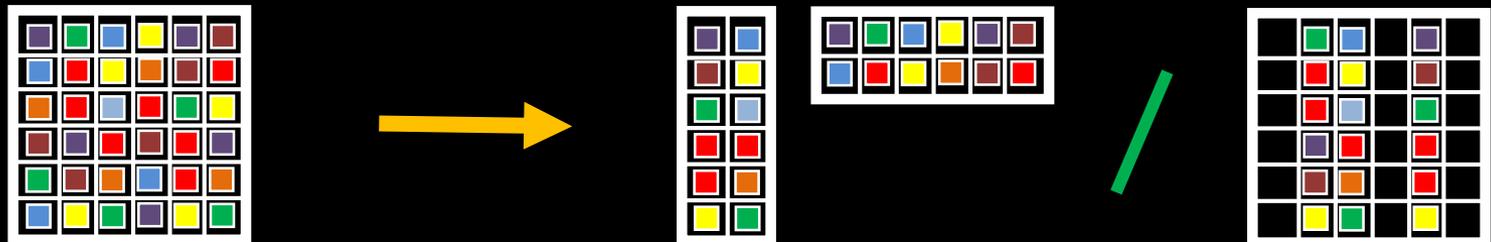
- Unstructured sparsity - CNNs
  - *Dataset*: ImageNet-1K
  - *Models*: ResNet50 & MobileNetV1
  - *Sparsity range*: 80 - 99%
    - Ultra-sparse regime: 98 - 99%
- Structured sparsity – Low rank in RNNs
  - *Datasets*: Google-12 (keyword spotting), HAR-2 (activity recognition)
  - *Model*: FastGRNN
- Additional
  - Transfer of learnt budgets to other sparsification techniques
  - STR for global, per-weight sparsity & filter/kernel pruning

# Unstructured vs Structured Sparsity

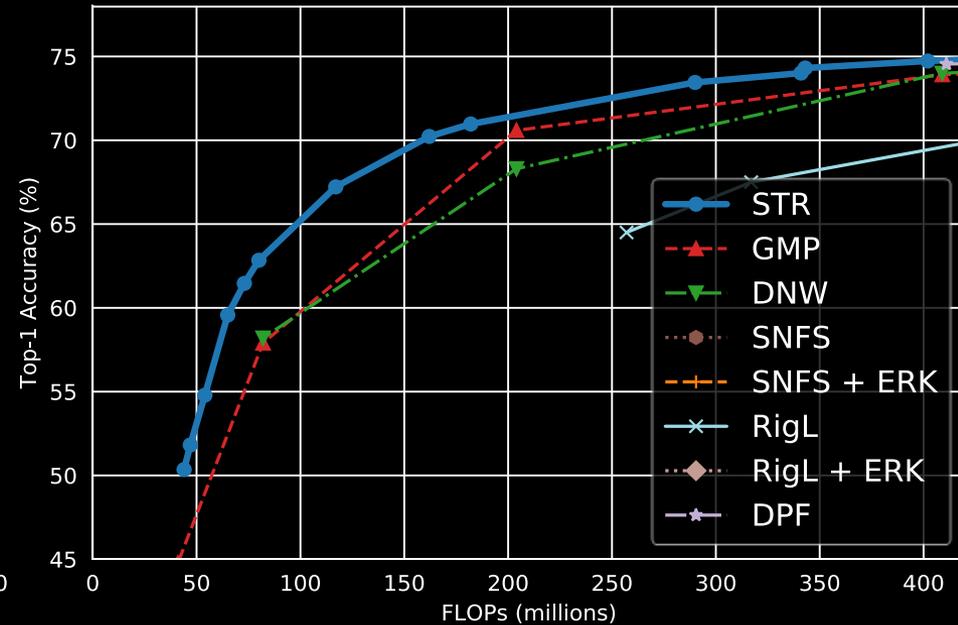
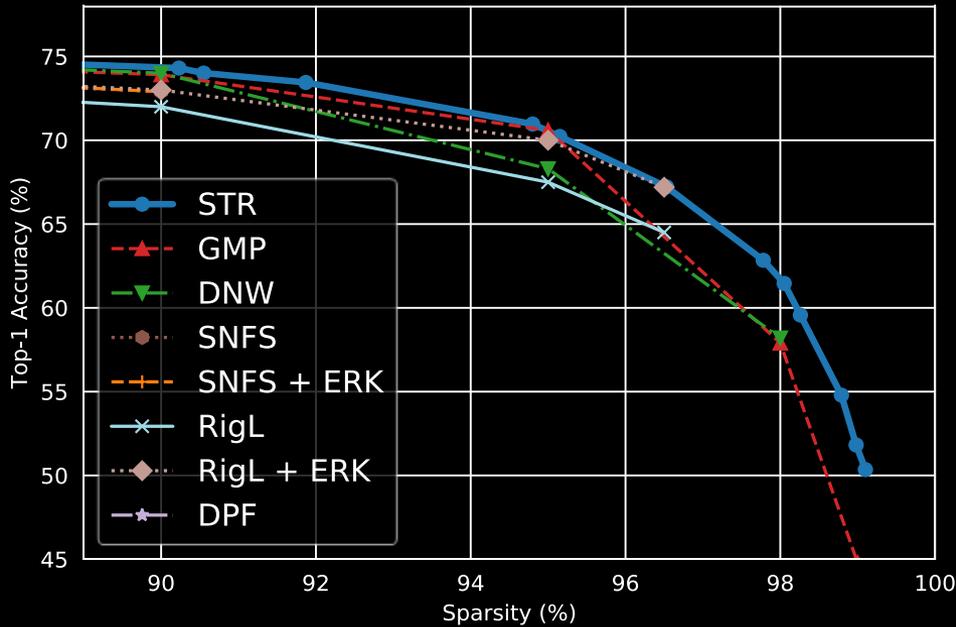
- Unstructured sparsity
  - Typically magnitude based pruning with global or layer-wise thresholds



- Structured sparsity
  - Low-rank & neuron/filter/kernel pruning

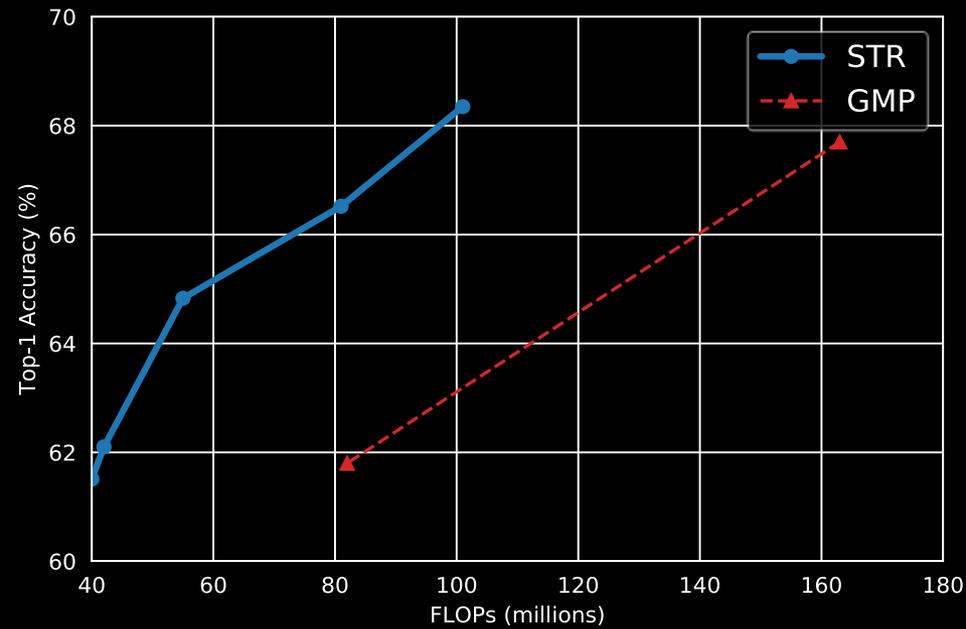
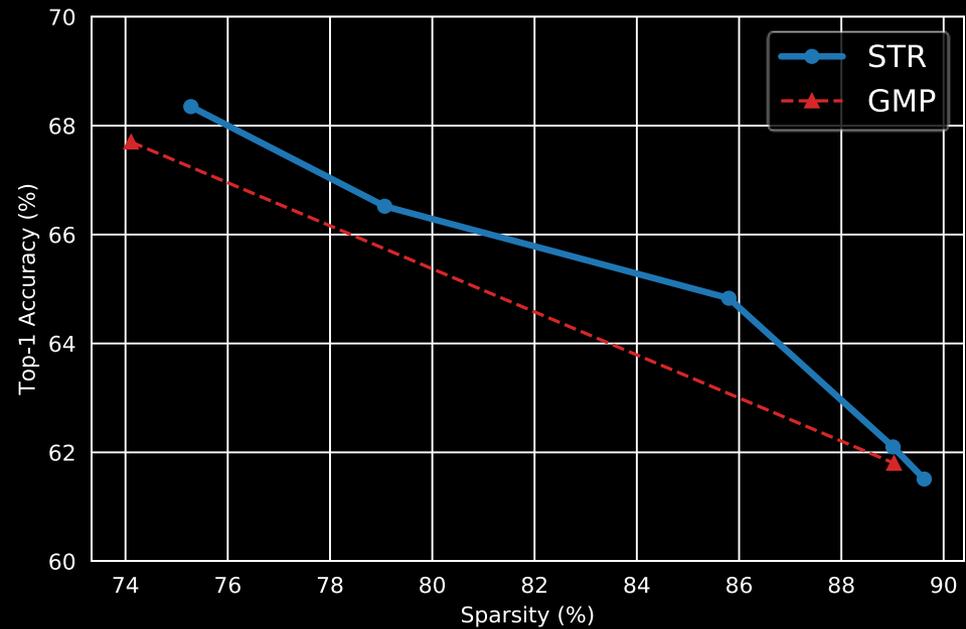


# STR Unstructured Sparsity: ResNet50



- STR requires 20% lesser FLOPs with same accuracy for 80-95% sparsity
- STR achieves 10% higher accuracy than baselines in 98-99% regime

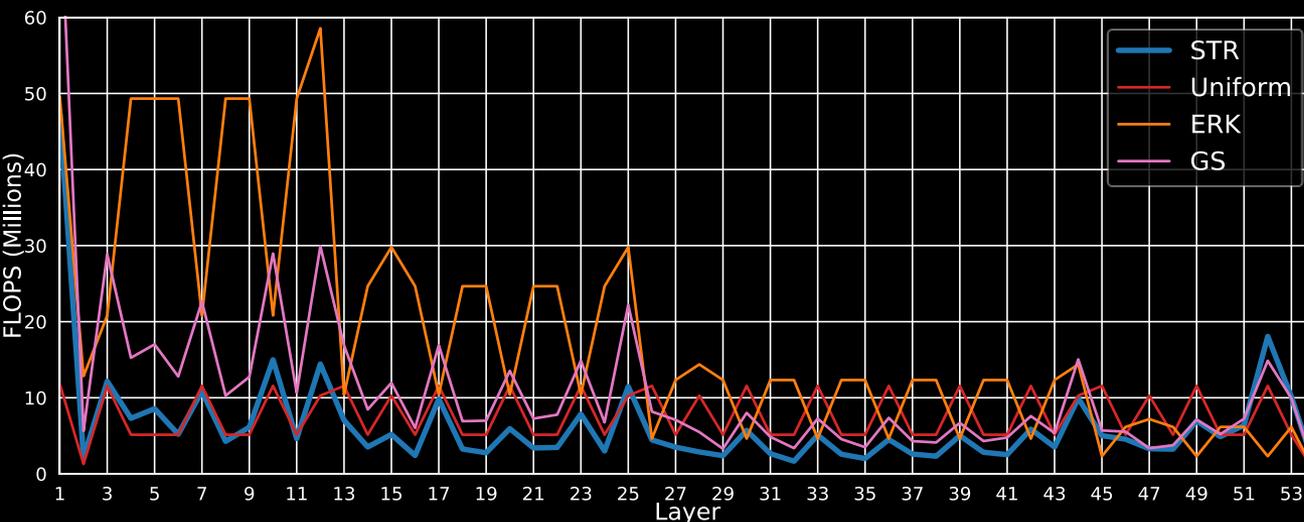
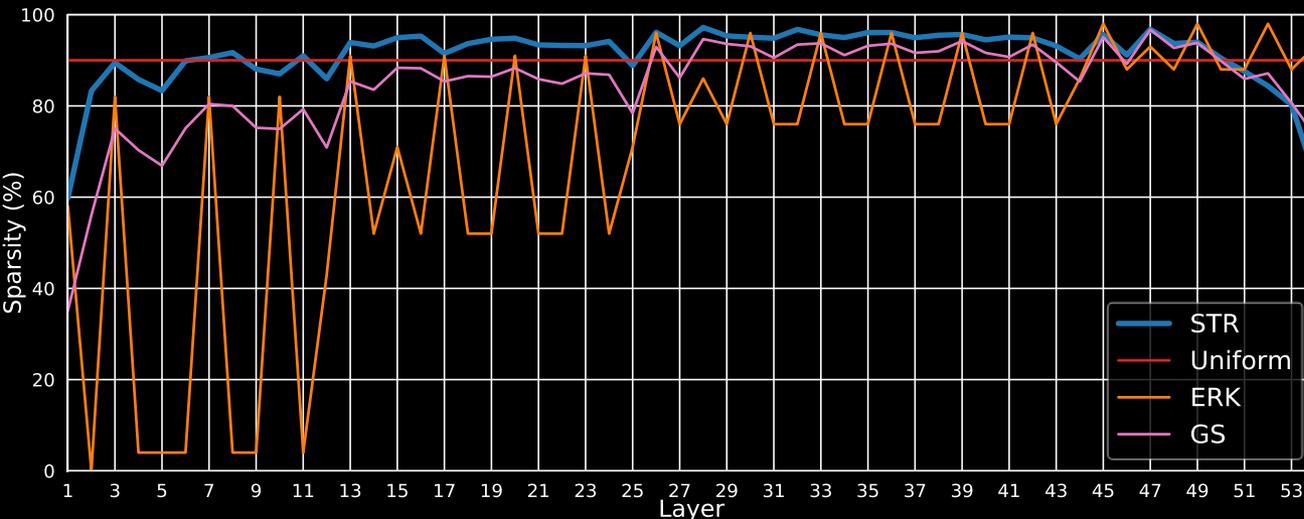
# STR Unstructured Sparsity: MobileNetV1



- STR maintains accuracy for 75% sparsity with 62M lesser FLOPs
- STR has ~50% lesser FLOPs for 90% sparsity with same accuracy

# STR Sparsity Budget: ResNet50

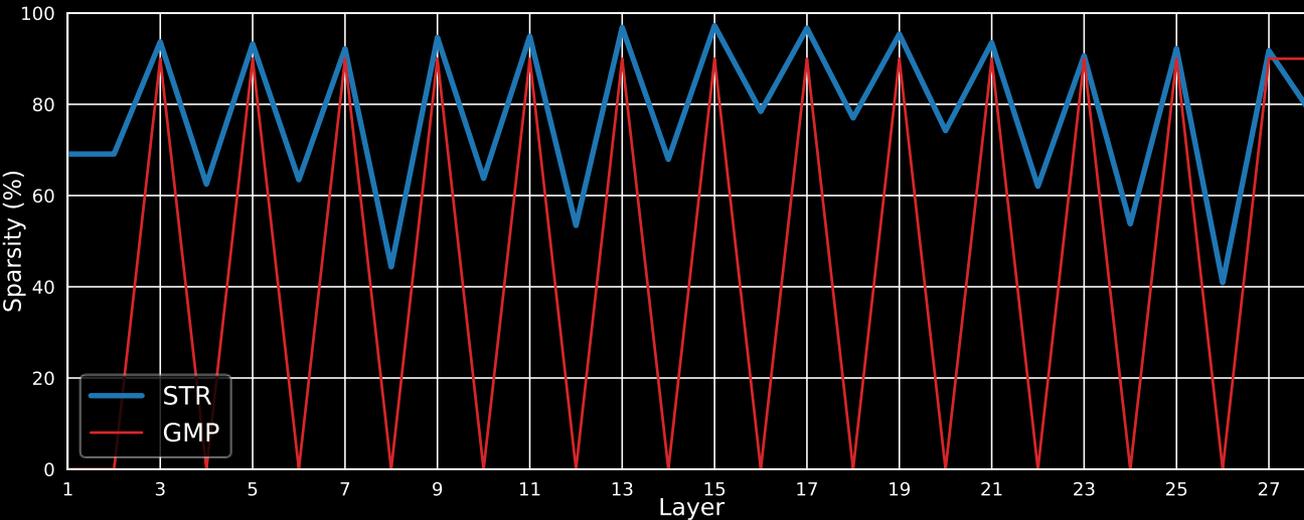
Layer-wise sparsity and FLOPs budgets for 90% sparse  
ResNet50 on ImageNet-1K



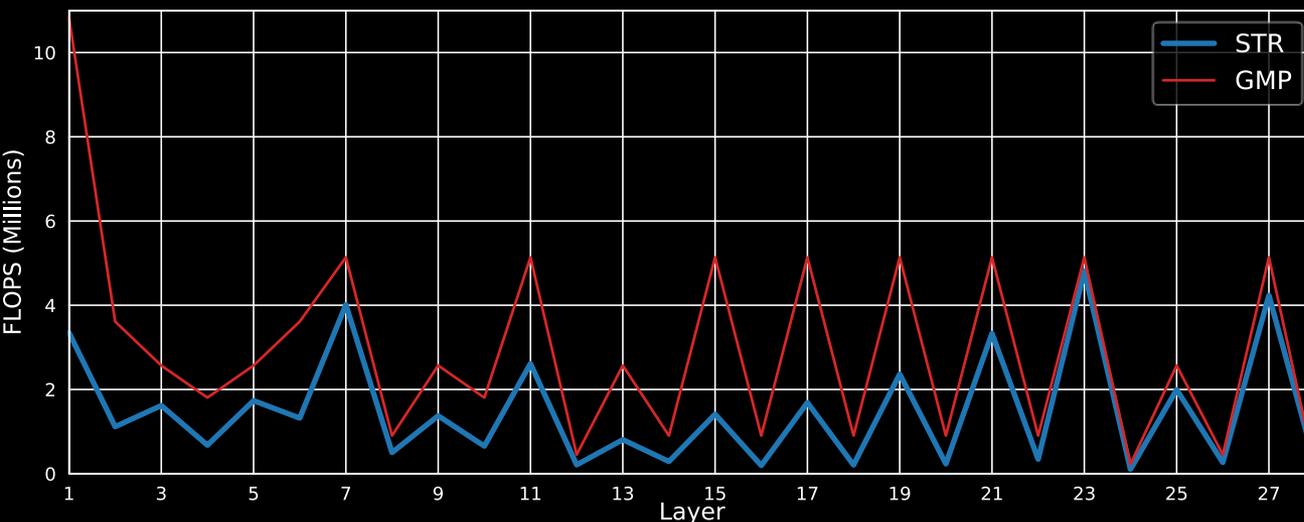
- STR learns sparser initial layers than the non-uniform sparsity baselines
- STR makes last layers denser than all baselines
- STR produces sparser backbones for transfer learning
- STR adjusts the FLOPs across layers such that it has lower total inference cost than the baselines

# STR Sparsity Budget: MobileNetV1

Layer-wise sparsity and FLOPs budgets for 90% sparse  
MobileNetV1 on ImageNet-1K



- STR automatically keeps depth-wise separable conv layers denser than rest of the layers



- STR's budget results in 50% lesser FLOPs than GMP

# STRConv

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**Algorithm 1** PyTorch code for STRConv with per-layer threshold.

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```
import torch
import torch.nn as nn
import torch.nn.functional as F

from args import args as parser_args

def softThreshold(x, s, g=torch.sigmoid):
    # STR on a weight x (can be a tensor) with "s" (typically a scalar, but can be a tensor) with function "g".
    return torch.sign(x)*torch.relu(torch.abs(x)-g(s))

class STRConv(nn.Conv2d): # Overloaded Conv2d which can replace nn.Conv2d
    def __init__(self, *args, **kwargs):
        super().__init__(*args, **kwargs)
        # "g" can be chosen appropriately, but torch.sigmoid works fine.
        self.g = torch.sigmoid
        # parser_args gets arguments from command line. sInitValue is the initialization of "s" for all layers. It
        # can take in different values per-layer as well.
        self.s = nn.Parameter(parser_args.sInitValue*torch.ones([1, 1]))
        # "s" can be per-layer (a scalar), global (a shared scalar across layers), per-channel/filter (a vector)
        # or per individual weight (a tensor of the size self.weight). All the experiments use per-layer "s" (a
        # scalar) in the paper.

    def forward(self, x):

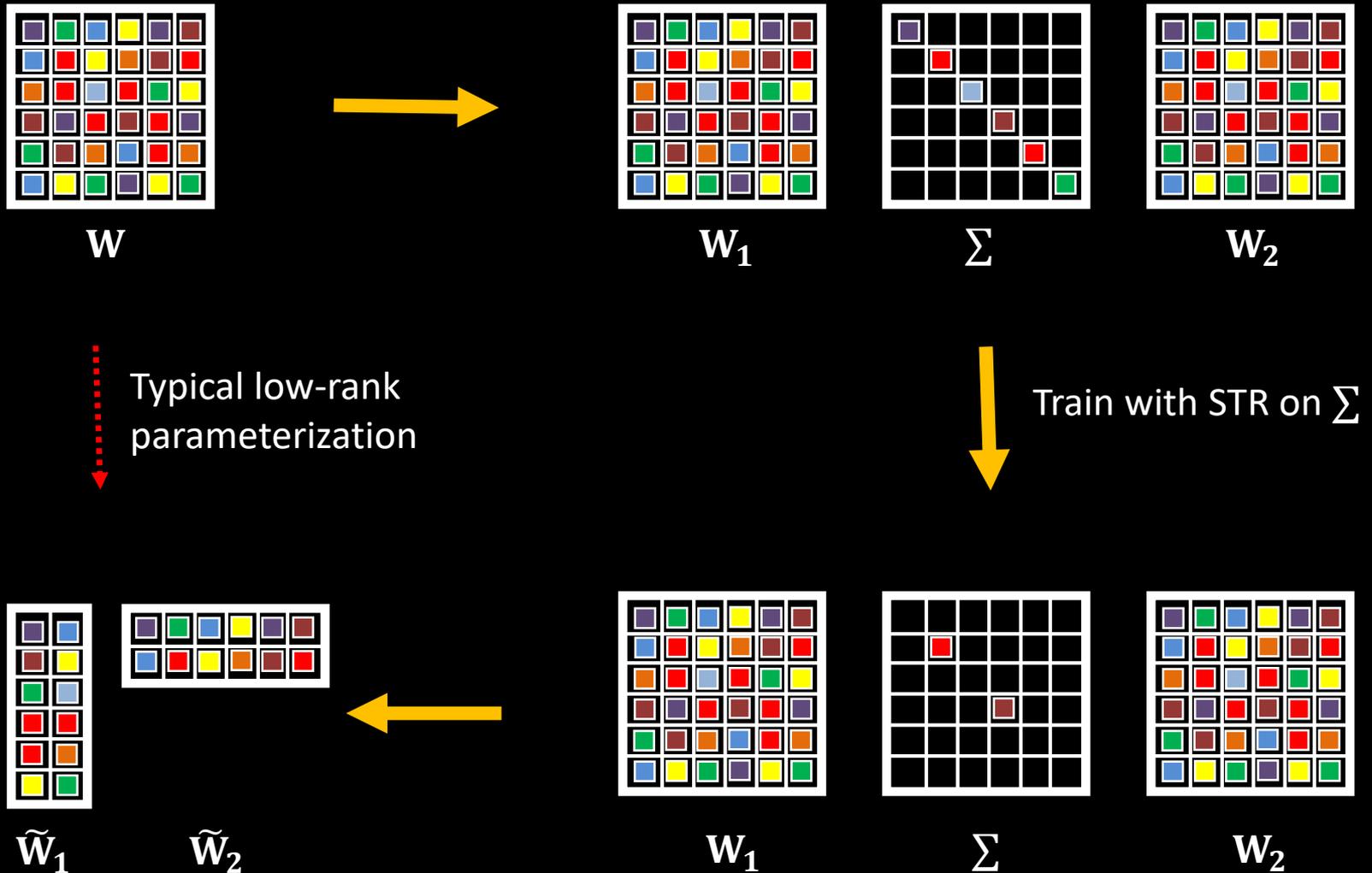
        self.sparseWeight = softThreshold(self.weight, self.s, self.g)
        # Parameters except "x" and "self.sparseWeight" can be chosen appropriately. All the experiments use
        # default PyTorch arguments.
        x = F.conv2d(x, self.sparseWeight, self.bias, self.stride, self.padding, self.dilation, self.groups)

        return x

# FC layer is implemented as a 1x1 Conv2d and STRConv is used for FC layer as well.
```

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# STR Structured Sparsity: Low rank



# STR – Critical Design Choices

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- Weight-decay  $\lambda$ 
  - Controls overall sparsity
  - Larger  $\lambda \rightarrow$  higher sparsity at the cost of some instability
- Initialization of  $s_l$ 
  - Controls finer sparsity exploration
  - Controls duration of dense training
- Careful choice of  $g(\cdot)$ 
  - Drives the training dynamics
  - Better functions which consistently revive dead weights

# STR - Conclusions

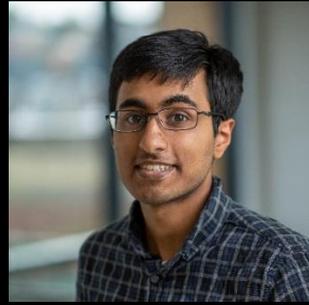
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- STR enables stable end-to-end training (with no additional cost) to obtain sparse & accurate DNNs
- STR efficiently learns per-layer sparsity budgets
  - Reduces FLOPs by up to 50% for 80-95% sparsity
  - Up to 10% more accurate than baselines for 98-99% sparsity
  - Transferable to other sparsification techniques
- Future work
  - Formulation to explicitly minimize FLOPs
  - Stronger guarantees in standard sparse regression setting
- Code, pretrained models and sparsity budgets available at

<https://github.com/RAIVNLab/STR>



Aditya



Vivek\*



Raghav\*



Mitchell\*

# Thank You

Prateek



Sham



Ali

