Efficient Full-Matrix Adaptive Regularization

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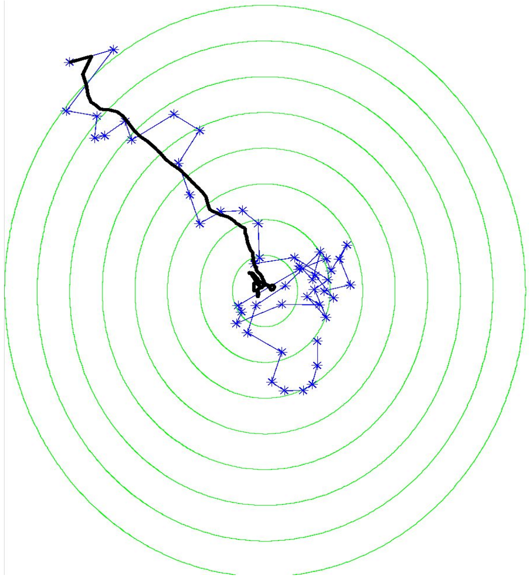
Adaptive Preconditioning in ML

- Optimization in ML: training neural nets → minimizing non-convex losses

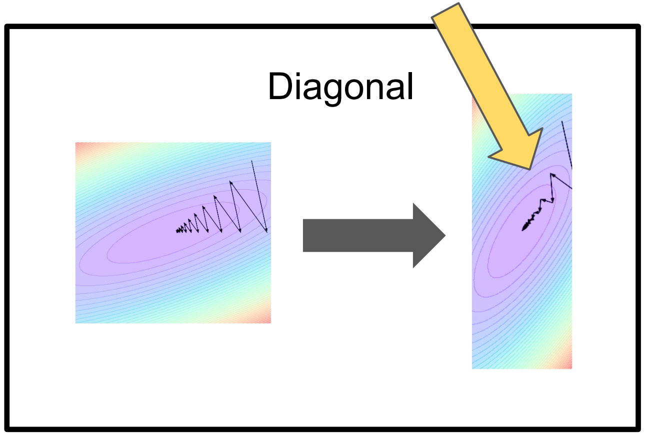
- **Diagonal** Adaptive Optimizers: each coordinate has a different learning rate according to past gradients
  - AdaGrad, Adam, RMSProp
  - Works well in practice

Theory is only known for convex losses at the time



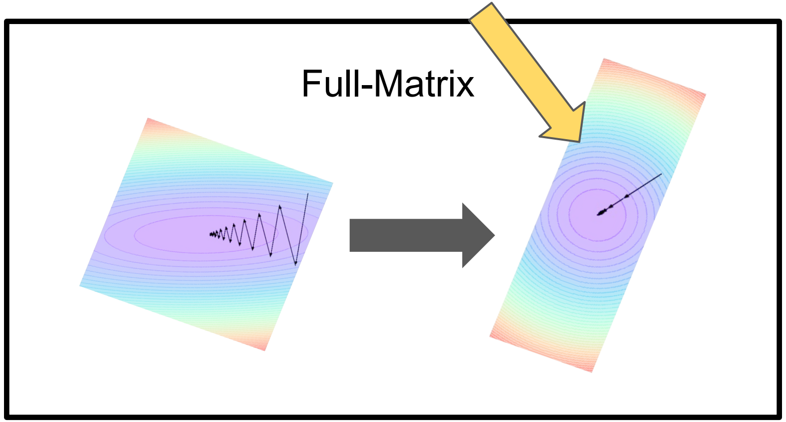
Adaptive Preconditioning: Intuition

Doesn't adapt to a rotated basis



$$x_{t+1} \leftarrow x_t - diag\left[\sum_{s=1}^{t} g_s g_s^{\top}\right]^{-1/2} \cdot g_t$$

Learns the correct basis, faster optimization



Expensive!

$$x_{t+1} \leftarrow x_t - \left[\sum_{s=1}^{t} g_s g_s^{\top}\right]^{-1/2} \cdot g_t$$

Can we have a linear time algorithm?

Our Results

- **GGT:** a new adaptive optimizer
  Efficient full-matrix (low-rank) AdaGrad

- **Experiments**: faster training and sometimes better generalization on vision and language tasks
- GPU-friendly Implementation

- **Theory**: “adaptive” convergence rate on convex and non-convex functions
- **Up to $O(1/\sqrt{d})$ faster than SGD**

The GGT Trick

- **Scalar Case:**

$$(a \times a)^{-1/2} = a \times (a \times a)^{-3/2} \times a$$

- **Matrix Case:**

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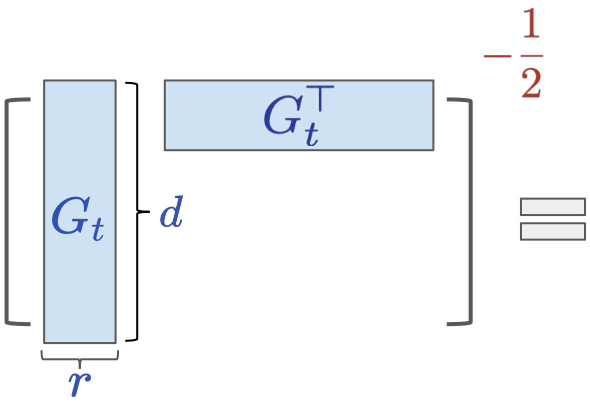

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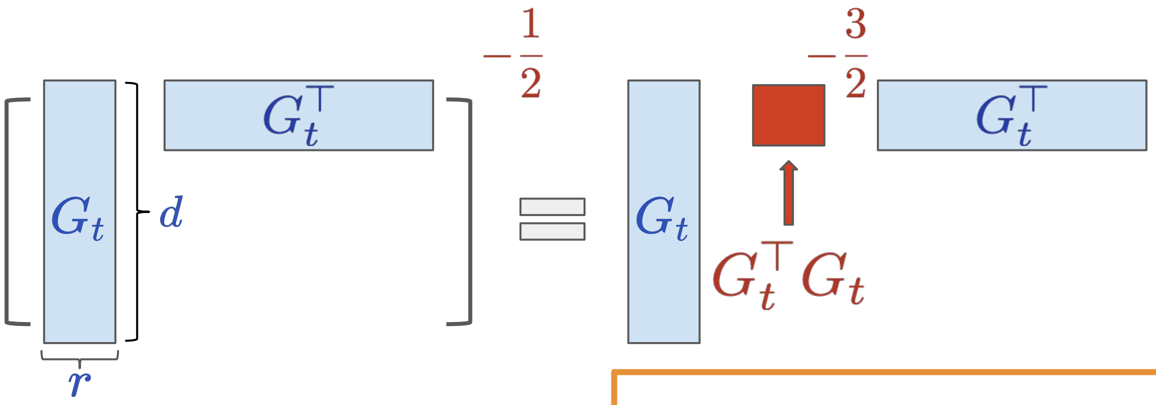
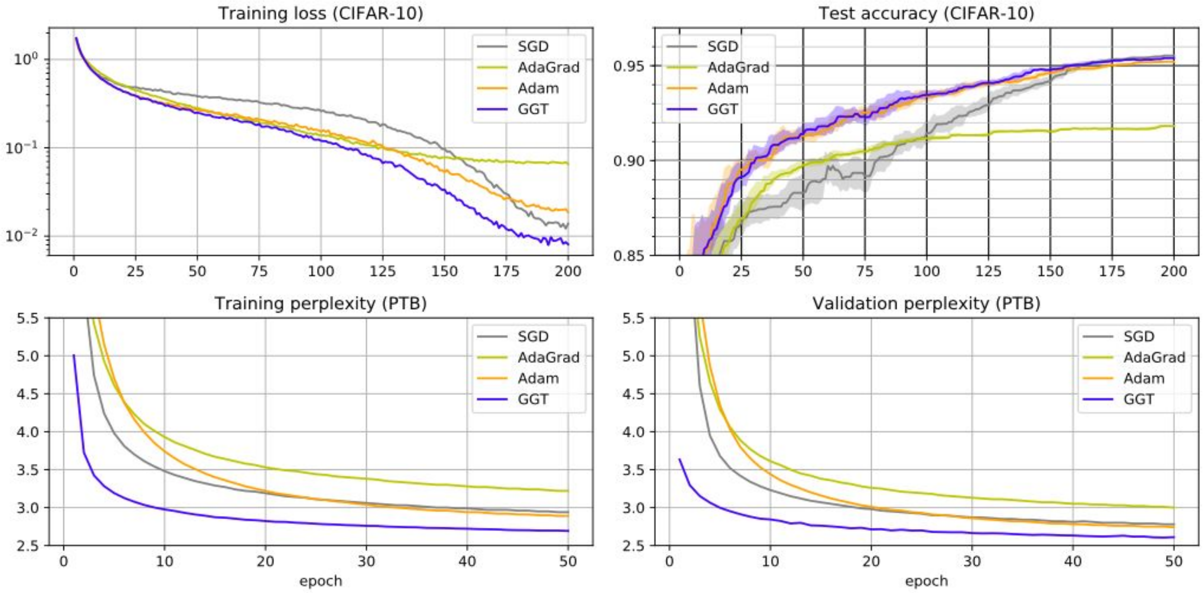


Efficient implementation on the GPU!

Large-Scale Experiments (CIFAR-10, PTB)



- Resnet-26 for CIFAR-10 and LSTM for PTB
- Better and faster training
- Initial acceleration in optimizing the LSTM
- Better validation ppl for the LSTM

Theory

- Define the *adaptivity ratio*:

$$\mu^2 = \frac{\text{AdaGrad Regret}}{\text{worst-case OGD Regret}}$$

[DHS10]: $\mu^2 \in [\frac{1}{\sqrt{d}}, \sqrt{d}]$ for diagonal AdaGrad, sometimes smaller for full-matrix AdaGrad

- **Non-Convex reduction**: GGT* converges in $\tilde{O}(\frac{\mu^2\sigma^2}{\epsilon^4})$ steps

- First step towards analyzing adaptive methods in non-convex optimization

* Idealized modification of GGT for analysis. See paper for details.

A note on the important parameters

- Improving dependence on epsilon: $\frac{1}{\epsilon^4} \to \frac{1}{\epsilon^{3.5}}$

  In practice $\epsilon \sim 0.1$, leading to an improvement of about 3.1

- Instead our improvement can be as large as the dimension, which can be 1e7 for language models

- **Huge untapped potential for large-scale optimization!**

Thank You!

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