Predicate Exchange

Inference with Declarative Knowledge

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Objective

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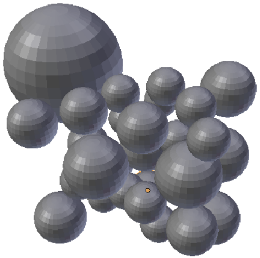
Objective: Given a probabilistic simulator π and predicate ℓ on the output of π, sample from posterior $p(\pi \mid \ell \text{ is true})$.

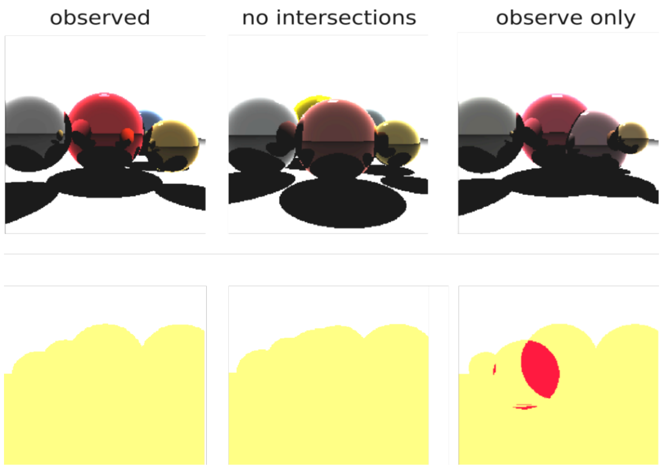
Priors with constraints

condition on balls not intersecting



Inverse Graphics with constraints

condition on balls not intersecting



Predicate Exchange

Predicate Exchange: An inference prodecure which samples from models conditioned on predicates, through two steps:

(i) **Predicate Relaxation** constructs a soft predicate $\tilde{\ell}$ from ℓ. $\tilde{\ell}$ maps $\mathbf{x}$ to a value in a continuous Boolean algebra: the unit interval $[0, 1]$ with continuous logical connectives $\tilde{\wedge}$, $\tilde{\vee}$ and $\tilde{\neg}$.

(i) Soft equality $x \tilde{=} y$: $k_\alpha(\rho(x, y))$
(ii) Soft inequality $x \tilde{>} y$: $k_\alpha(\rho(x, [y, \infty]))$
(iii) Soft conjunction $\tilde{\wedge}$: $\max(x, y)$
(iv) Soft disjunction $\tilde{\vee}$: $\min(x, y)$

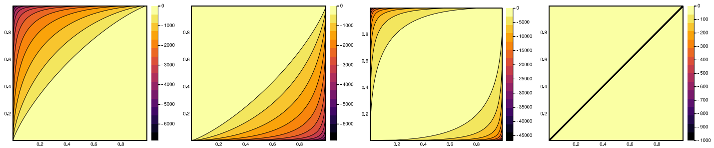


Figure 1:log of $x \tilde{>} y$, $x \tilde{<} y$, $x \tilde{=} y$, and $\tilde{\neg}(x \tilde{=} y)$

Convert predicates into soft predicates

Soft predicate represents degree to which hard predicate is satisfied

$$(x > y) \vee \neg(x^2 = 2) \rightarrow (x \mathrel{\tilde{>}} y) \mathbin{\tilde{\vee}} \tilde{\neg}(x^2 \mathrel{\tilde{=}} 2)$$

Approximate Posterior

Assuming a prior density p, the approximate posterior f is the product:

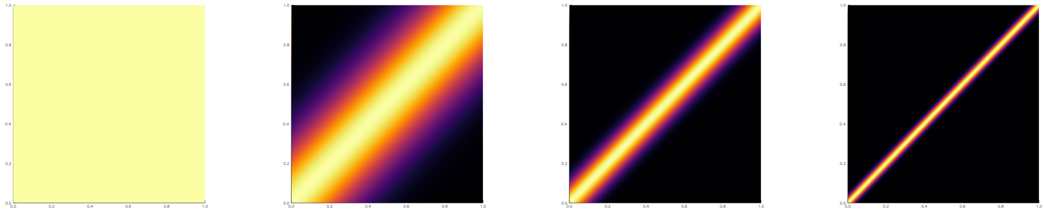
$$f(\mathbf{x}) = p(\mathbf{x}) \cdot \tilde{\ell}(\mathbf{x})$$

Example: if $X_{1,2} \sim \mathcal{N}(0,1)$ is conditioned on $X_1 + X_2 = 0$, aprx posterior:

$$f_\alpha(x_1, x_2) = \mathcal{N}_{0,1}(x_1) \cdot \mathcal{N}_{0,1}(x_2) \cdot k_\alpha(\rho(x_1 + x_2, 0))$$

Temperature trades-off accuracy / convergence

$\alpha = 0$ $\alpha = 10$ $\alpha = 100$ $\alpha = 1000$



Replica Exchange

(ii) **Replica Exchange** is a MCMC method that simulates several replicas conditioned model at different temperatures

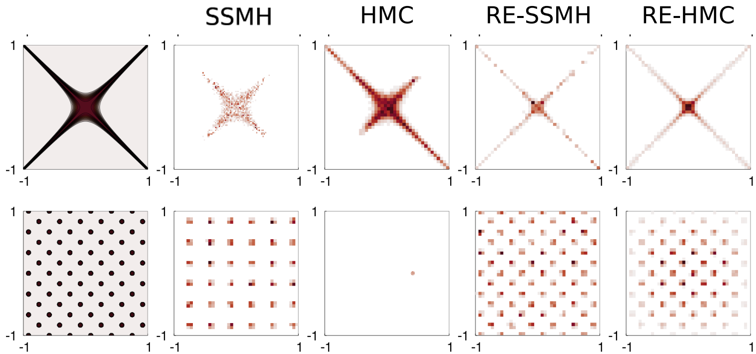
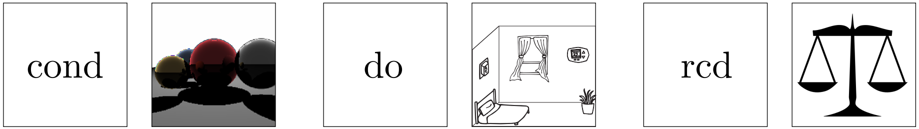


Figure 2:Samples from single site MH, Hamiltonian Monte Carlo, and replica exchange

Omega.jl: A Causal, Higher-Order PPL

github.com/zenna/Omega.jl

Poster #52



conditional inference

causal inference

higher-order inference