Non-monotone Submodular Maximization with Nearly Optimal Adaptivity and Query Complexity

Matthew Fahrbach[1] Vahab Mirrokni[2] Morteza Zadimoghaddam[2]

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[1]Georgia Tech [2]Google

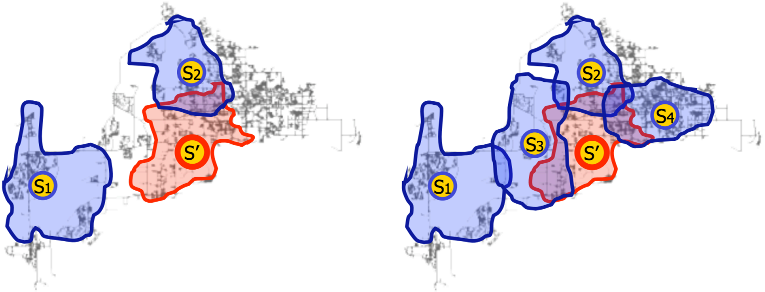
Submodular Functions

Def. A function $f : 2^N \to \mathbb{R}$ is submodular if for all $S \subseteq T \subseteq N$ and $x \in N \setminus T$ we have

$$f(S \cup \{x\}) - f(S) \geq f(T \cup \{x\}) - f(T).$$

- Models the property of diminishing returns

Example. $f(S)$ is the coverage of placing sensors at locations S.



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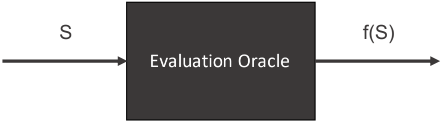
Applications in machine learning.

- Document summarization
- Exemplar clustering
- Feature selection
- Graph cuts

Assumption. Evaluation oracle that returns $f(S)$ in $O(1)$ time.



Problem. Maximize $f(S)$ such that $|S| \leq k$ using a small number of adaptive rounds and oracle queries.

Adaptivity Complexity

Def. The adaptivity of a distributed algorithm is the minimum needed round complexity, where in each round the algorithm can make poly(n) independent queries to the value oracle.

- Rank of partial order on queries ordered by dependence
- Models communication complexity with oracle

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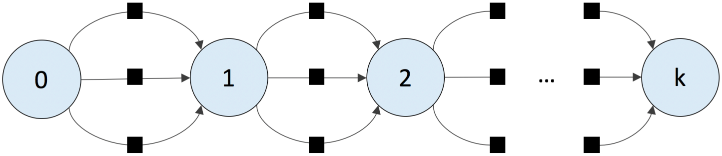
Example. Greedy algorithm for constrained maximization

1. Set $S_0 \leftarrow \emptyset$
2. For $i = 1$ to k:
3. Set $S_i \leftarrow S_{i-1} \cup \{\arg\max_{x \in N} f(S_{i-1} \cup \{x\})\}$



Main Results

Problem. Submodular maximization of a non-monotone function subject to a cardinality constraint k

Algorithm	Approximation	Adaptivity	Queries
BFS16	$1/e \approx 0.371$	$O(k)$	$O(n)$
BBS18 (NeurIPS 18)	0.183	$O(\log^2(n))$	$\tilde{O}(\mathrm{OPT}^2 n)$
CQ19 (STOC 19)	0.172	$O(\log^2(n))$	$\tilde{O}(nk^4)$
ENV19 (STOC 19)	0.371	$O(\log^2(n))$	$\tilde{O}(nk^2)$
FMZ19	0.039	$O(\log(n))$	$O(n \log(k))$

Adaptivity Hardness [BS18]. We need $\Omega(\log(n)/\log\log(n))$ adaptive rounds to achieve a constant-factor approximation.