# Classification from Positive, Unlabeled and Biased Negative Data

Poster #180

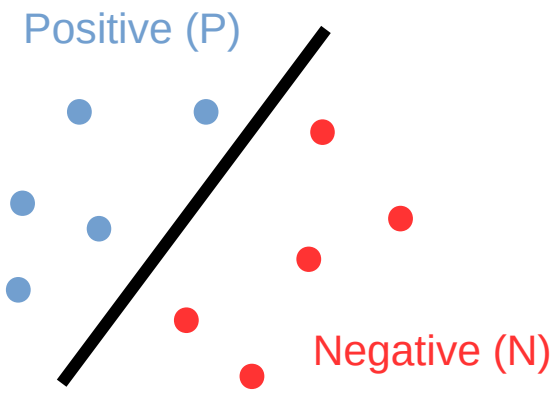
**Yu-Guan Hsieh**[1], Gang Niu[2], Masashi Sugiyama[2,3]

[1] ENS Paris, France [2] RIKEN, Japan [3] The University of Tokyo, Japan

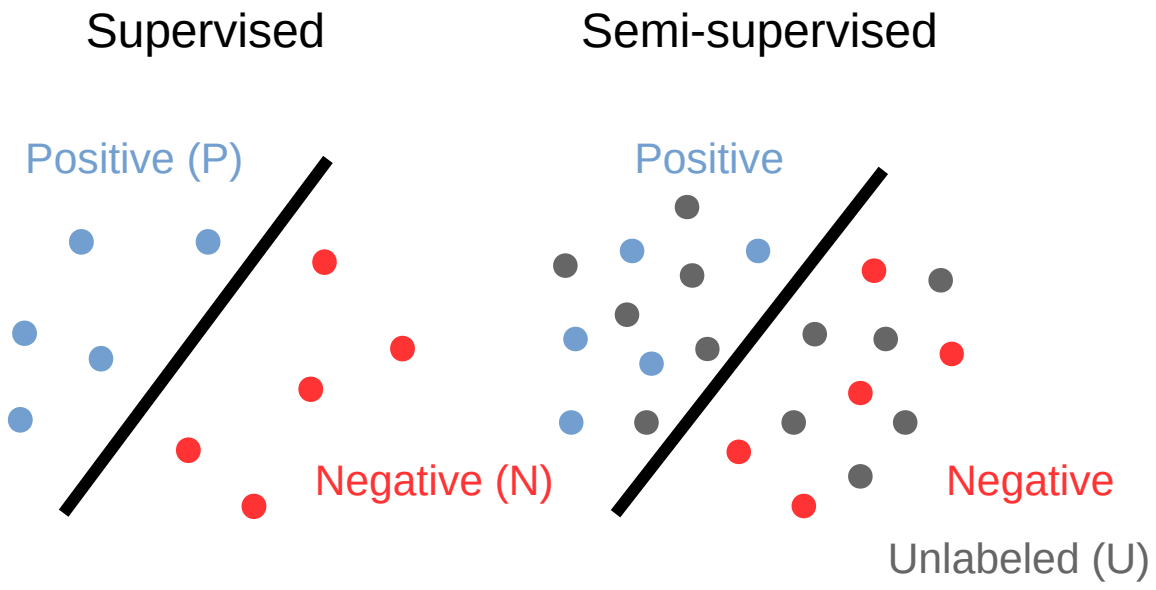
# Background and problem setup

# Background and problem setup

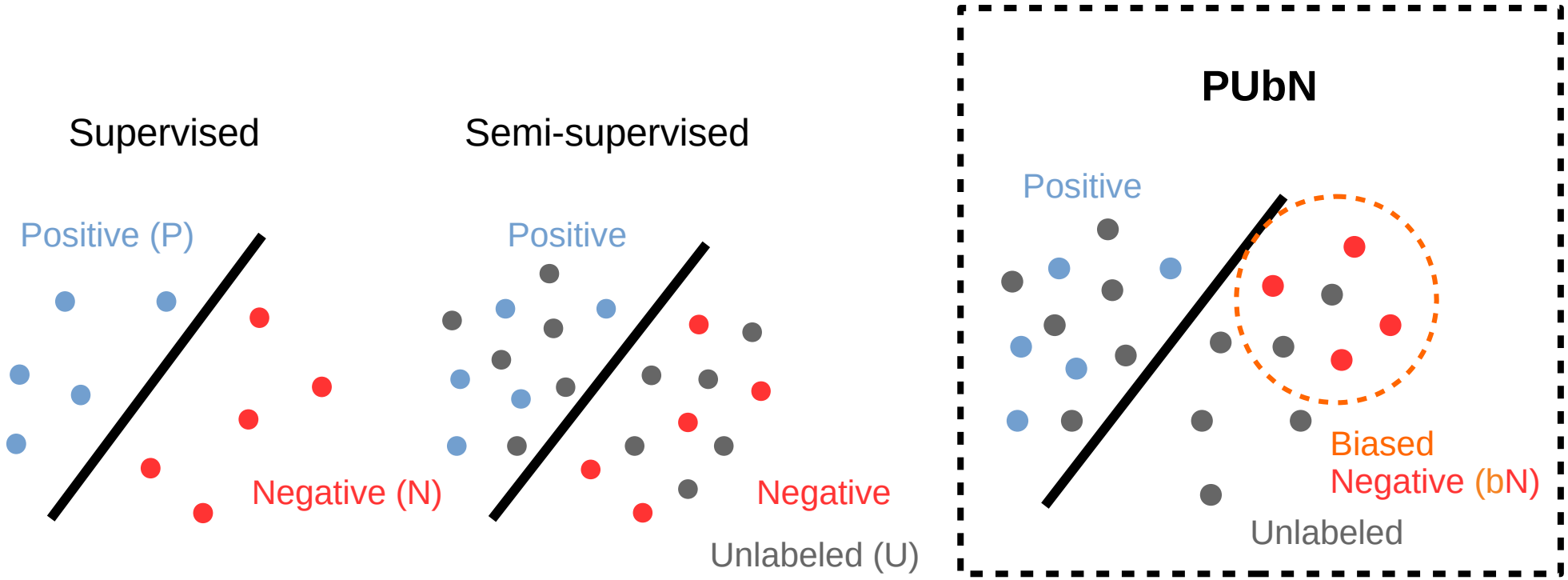
Supervised



# Background and problem setup

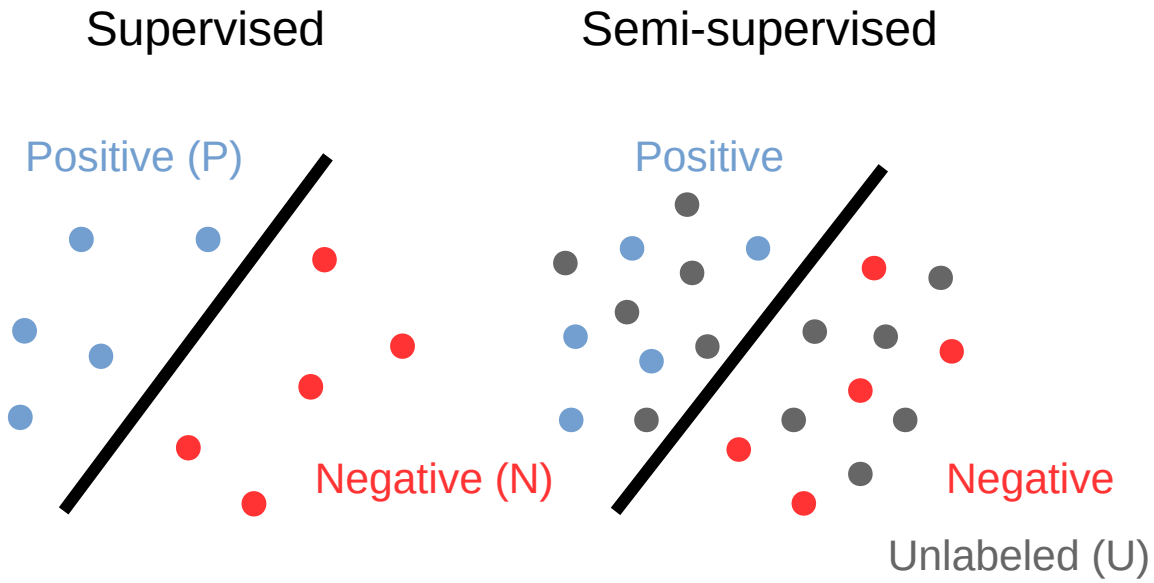


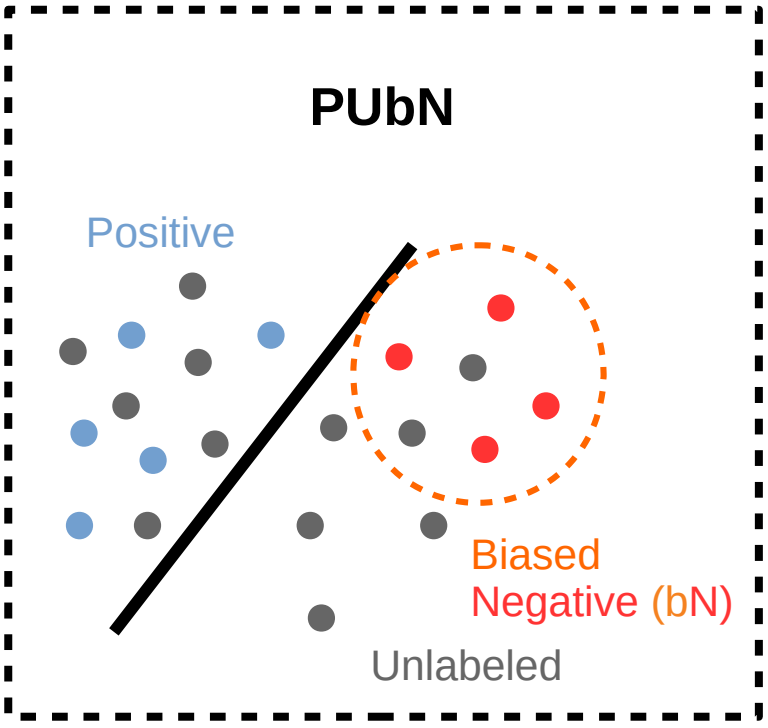
# Background and problem setup



Poster #180

# Background and problem setup



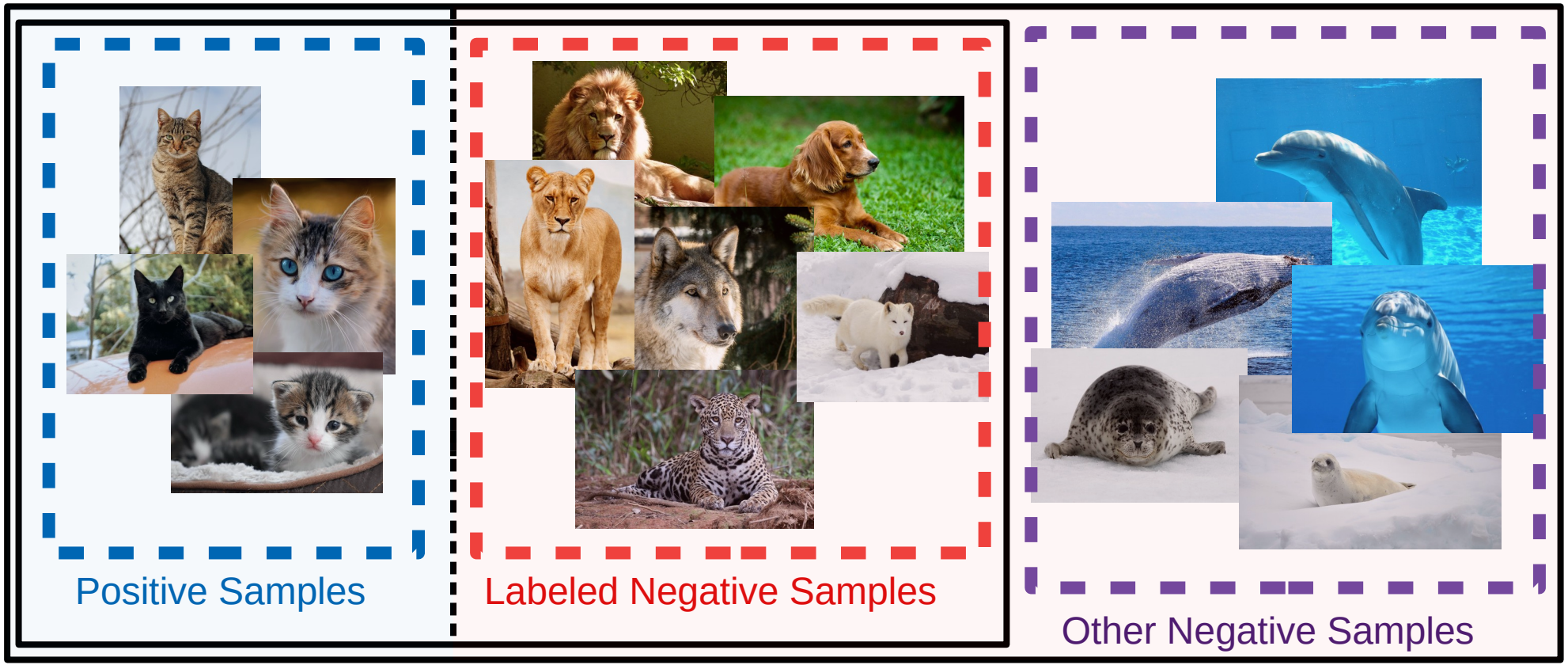




- $\sim p_{\mathrm{P}}(x) := p(x \mid y = +1)$
- $\sim p_{\mathrm{N}}(x) := p(x \mid y = -1)$
- $\sim p(x)$
- $\sim p_{\mathrm{bN}}(x) := p(x \mid y = -1, s = +1)$

$y = +1 \Rightarrow s = +1$

# Motivating examples






- Information retrieval, text classification, sentiment analysis
- Medical diagnosis: healthy population requesting physical exams is biased

# Method: Empirical risk estimator

$$\min_{g \in \mathcal{G}} \underbrace{\mathbb{E}_{(x,y)\sim p(x,y)}[\ell(yg(x))]}_{R(g)}$$

Risk Minimization

Unbiased
Estimator

$$\min_{g \in \mathcal{G}} \underbrace{\frac{1}{n}\sum_{i=1}^{n} \ell(y_i g(x_i))}_{\hat{R}(g)}$$

Unbiased labeled data
Empirical Risk Minimization

# Method: Empirical risk estimator

$$R(g) = \pi \mathbb{E}_{x \sim p_{\mathrm{P}}(x)}[\ell(g(x))] + \pi \mathbb{E}_{x \sim p_{\mathrm{P}}(x)} \left[ 1_{\sigma(x) > \eta} \, \ell(-g(x)) \frac{1 - \sigma(x)}{\sigma(x)} \right]$$

$$+ \rho \mathbb{E}_{x \sim p_{\mathrm{bN}}(x)}[\ell(-g(x))] + \rho \mathbb{E}_{x \sim p_{\mathrm{bN}}} \left[ 1_{\sigma(x) > \eta} \, \ell(-g(x)) \frac{1 - \sigma(x)}{\sigma(x)} \right]$$

$$+ \mathbb{E}_{x \sim p(x)}[1_{\sigma(x) \leq \eta} \, \ell(-g(x))(1 - \sigma(x))]$$

σ(x) = p(s=+1|x) probability of x being labeled

η>0 determining how much we rely on the U data to approximate the risk

# Method: Empirical risk estimator

#P data

$$R(g) = \pi \mathbb{E}_{x \sim p_{\mathrm{P}}(x)}[\ell(g(x))] + \pi \mathbb{E}_{x \sim p_{\mathrm{P}}(x)}\left[1_{\sigma(x)>\eta}\, \ell(-g(x)) \frac{1-\sigma(x)}{\sigma(x)}\right]$$

$$+ \rho \mathbb{E}_{x \sim p_{\mathrm{bN}}(x)}[\ell(-g(x))] + \rho \mathbb{E}_{x \sim p_{\mathrm{bN}}}\left[1_{\sigma(x)>\eta}\, \ell(-g(x)) \frac{1-\sigma(x)}{\sigma(x)}\right]$$

#bN data

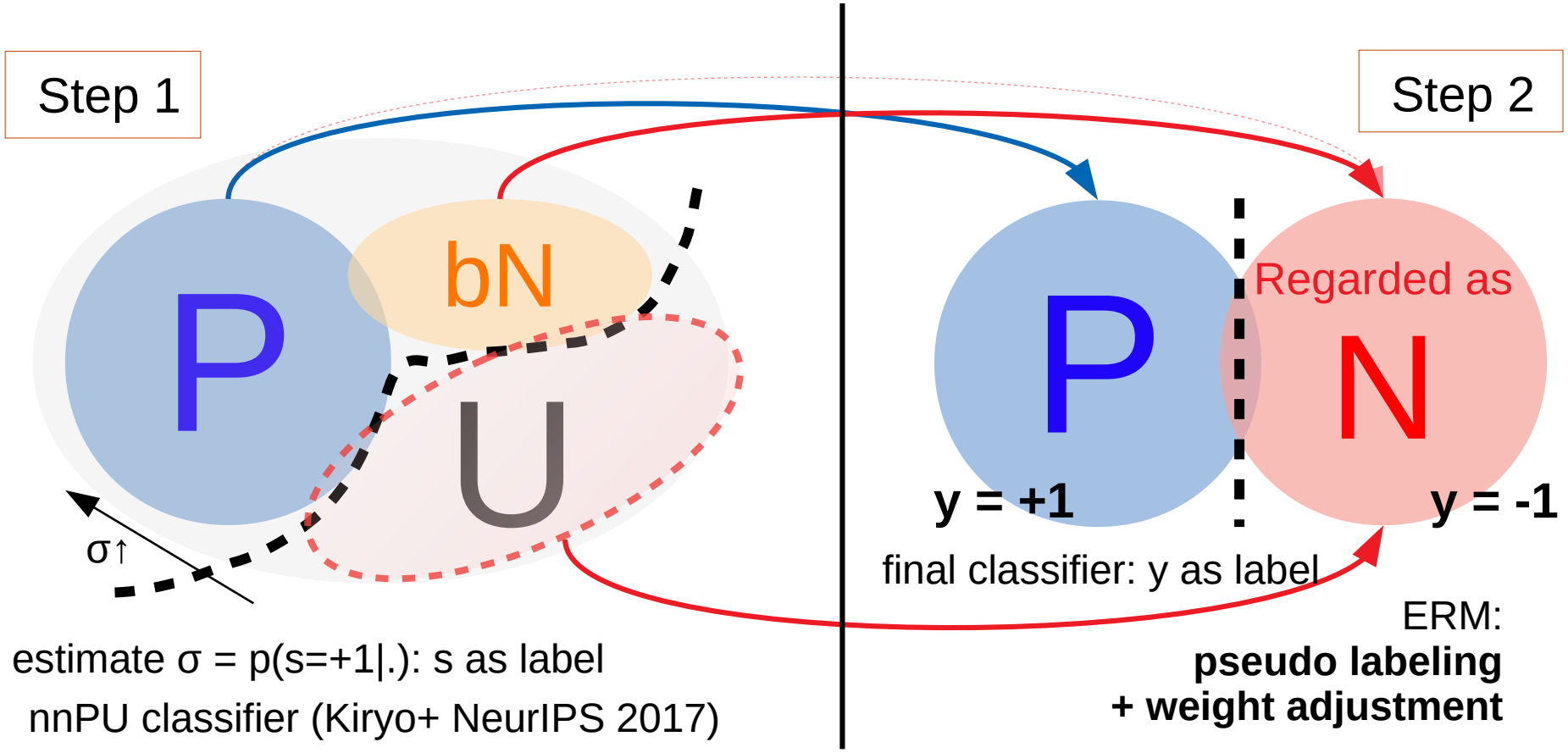
$$+ \mathbb{E}_{x \sim p(x)}[1_{\sigma(x)\leq\eta}\, \ell(-g(x))(1-\sigma(x))]$$

#U data

σ(x) = p(s=+1|x) probability of x being labeled

η>0 determining how much we rely on the U data to approximate the risk

# Method: Illustration



# Estimation error bound

With probability at least 1-δ

$$R(\hat{g}) - R(g^*) \leq \frac{4\pi L_\ell}{\eta}\mathfrak{R}_{n_\mathsf{P},p_\mathsf{P}}(\mathcal{G}) + \frac{2\pi C_\ell}{\eta}\sqrt{\frac{\ln(6/\delta)}{2n_\mathsf{P}}} + \frac{4\rho L_\ell}{\eta}\mathfrak{R}_{n_\mathsf{bN},p_\mathsf{bN}}(\mathcal{G}) + \frac{2\rho C_\ell}{\eta}\sqrt{\frac{\ln(6/\delta)}{2n_\mathsf{bN}}}$$

#P data

#bN data

$$+ 4L_\ell\mathfrak{R}_{n_\mathsf{U},p}(\mathcal{G}) + 2C_\ell\sqrt{\frac{\ln(6/\delta)}{2n_\mathsf{U}}} + 2C_\ell\sqrt{\zeta\epsilon} + \frac{2C_\ell}{\eta}\sqrt{(1-\zeta)\epsilon}$$

#U data

Bias due to inexact approximation of σ

# Experiments

Models: ConvNet / ResNet / FCN + Training: Amsgrad

| Dataset | P | π | bN | ρ | nnPU/nnPNU | PUbN(\N) | PU→PN |
|---|---|---|---|---|---|---|---|
| MNIST | 2, 4, 6, 8, 10 | 0.49 | Not given | NA | 5.76 ± 1.04 | **4.64 ± 0.62** | NA |
| | | | 1, 3, 5 | 0.3 | 5.33 ± 0.97 | 4.05 ± 0.27 | **4.00 ± 0.30** |
| | | | 9 > 5 > others | 0.2 | 4.60 ± 0.65 | 3.91 ± 0.66 | **3.77 ± 0.31** |
| CIFAR-10 | Airplane, automobile, ship, truck | 0.4 | Not given | NA | 12.02 ± 0.65 | **10.70 ± 0.57** | NA |
| | | | Cat, dog, horse | 0.3 | 10.25 ± 0.38 | **9.71 ± 0.51** | 10.37 ± 0.65 |
| | | | Horse > deer = frog > others | 0.25 | 9.98 ± 0.53 | **9.92 ± 0.42** | 10.17 ± 0.35 |
| CIFAR-10 | Cat, deer, dog, horse | 0.4 | Not given | NA | 23.78 ± 1.04 | **21.13 ± 0.90** | NA |
| | | | Bird, frog | 0.2 | 22.00 ± 0.53 | **18.83 ± 0.71** | 19.88 ± 0.62 |
| | | | Car, truck | 0.2 | 22.00 ± 0.74 | **20.19 ± 1.06** | 21.83 ± 1.36 |
| 20 Newsgroups | alt., comp., misc., rec. | 0.56 | Not given | NA | 14.67 ± 0.87 | **13.30 ± 0.53** | NA |
| | | | sci. | 0.21 | 14.69 ± 0.46 | **13.10 ± 0.90** | 13.58 ± 0.97 |
| | | | talk. | 0.17 | 14.38 ± 0.74 | **12.61 ± 0.75** | 13.76 ± 0.66 |
| | | | soc. > talk. > sci. | 0.1 | 14.41 ± 0.76 | **12.18 ± 0.59** | 12.92 ± 0.51 |