Gromov-Wasserstein Learning for Graph Matching and Node Embedding

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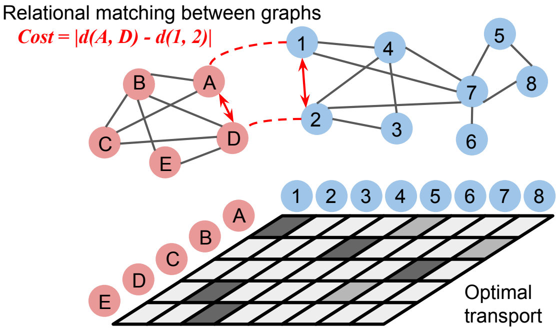
Problem Statement and Proposed Method

Given two graphs, we aim to achieve

- **Graph matching:** Finding a correspondence between their nodes.
- **Node embedding:** Embedding their nodes in the same space.

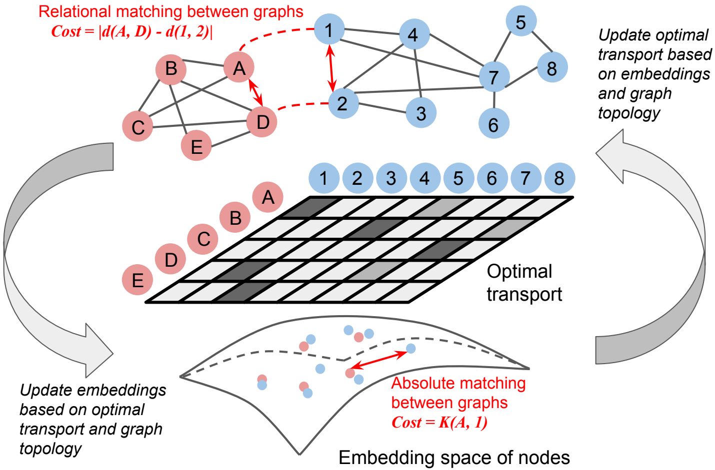
Unify them in our **Gromov-Wasserstein Learning (GWL)** framework.

$$d_{GW}(G_s, G_t) := \min_{\boldsymbol{T}\in\Pi(\boldsymbol{\mu}_s,\boldsymbol{\mu}_t)} \sum_{i,j,i',j'} L(c^s_{ij}, c^t_{i'j'}) T_{ii'} T_{jj'} = \min_{\boldsymbol{T}\in\Pi(\boldsymbol{\mu}_s,\boldsymbol{\mu}_t)} \langle \boldsymbol{L}(\boldsymbol{C}_s, \boldsymbol{C}_t, \boldsymbol{T}), \boldsymbol{T}\rangle.$$

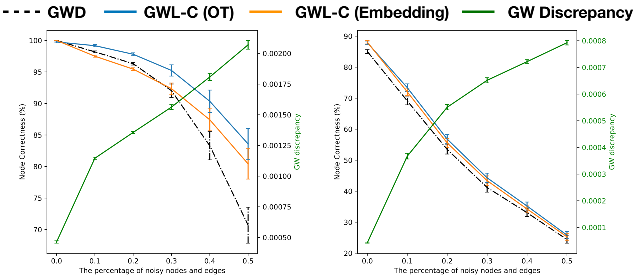


Gromov-Wasserstein Learning

$$\min_{\boldsymbol{X}_s,\boldsymbol{X}_t}\ \min_{\boldsymbol{T}\in\Pi(\boldsymbol{\mu}_s,\boldsymbol{\mu}_t)} \underbrace{\langle \boldsymbol{L}(\boldsymbol{C}_s(\boldsymbol{X}_s), \boldsymbol{C}_t(\boldsymbol{X}_t), \boldsymbol{T}), \boldsymbol{T}\rangle}_{\text{Gromov-Wasserstein discrepancy}} + \underbrace{\alpha\langle \boldsymbol{K}(\boldsymbol{X}_s, \boldsymbol{X}_t), \boldsymbol{T}\rangle}_{\text{Wasserstein discrepancy}} + \underbrace{\beta \sum\nolimits_{k=s,t} R(\boldsymbol{K}(\boldsymbol{X}_k, \boldsymbol{X}_k), \boldsymbol{C}_k)}_{\text{prior information}}.$$



Experimental Results



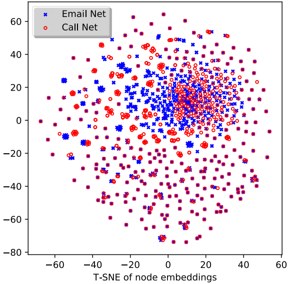


Table 1. Communication network matching results.

Method	Call→Email (Sparse) Node Correctness (%)	Call→Email (Dense) Node Correctness (%)
GAA	34.22	0.53
LRSA	38.20	2.93
TAME	37.39	2.67
GRAAL	39.67	0.48
MI-GRAAL	35.53	0.64
MAGNA++	7.88	0.09
HugAlign	36.21	3.86
NETAL	36.87	1.77
GWD	23.16±0.46	1.77±0.22
GWL-R	39.64±0.57	3.80±0.23
GWL-C	**40.45**±0.53	**4.23**±0.27