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Learning to optimize multigrid PDE solvers

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Solving PDEs is useful

- Predicting weather systems
- Aircraft and auto design
- Oceanic flow

Solving PDEs is hard

- High accuracy requires discretization on very fine grids
- Developing efficient solvers is an active research area since many decades ago
- Can we use machine learning to construct solvers?

Previous works

Learning to solve a single equation (new equation = retrain needed)

- Katrutsa et al, 2017: learning the prolongation for Poisson equation
- Hsieh, 2019: accelerate Poisson solvers
- Baque et al, 2018: simulate fluid dynamics
- Han et al, 2018: PDEs in high dimension
- ...

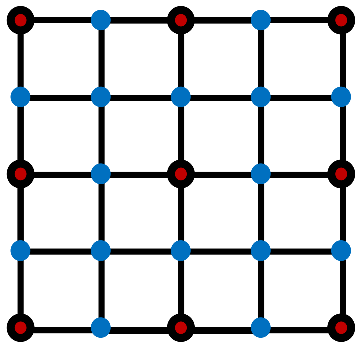
This work

- Learning how to solve a family of PDEs

- Example: 2D elliptic diffusion problems

$$-\nabla \cdot (g \nabla u) = f$$

- Focus on *multigrid solvers*
  - Solves the equation on multiple scales
  - Prolongation operator for moving between scales



Key elements of our approach

- Scope - *train a single network once for an entire class of PDEs*
- Unsupervised training - *no ground truth provided, and no equation is solved during training*
- Generalization - *train on small problems w. periodic BC & test on much larger problems w. Dirichlet BC*
- Efficient training – using Fourier analysis

TL;DR

- We pose the following learning problem

$$\min_{\theta} \mathbb{E}_{A\sim D}\, \rho\left(M\left(A, P_\theta(A)\right)\right)$$

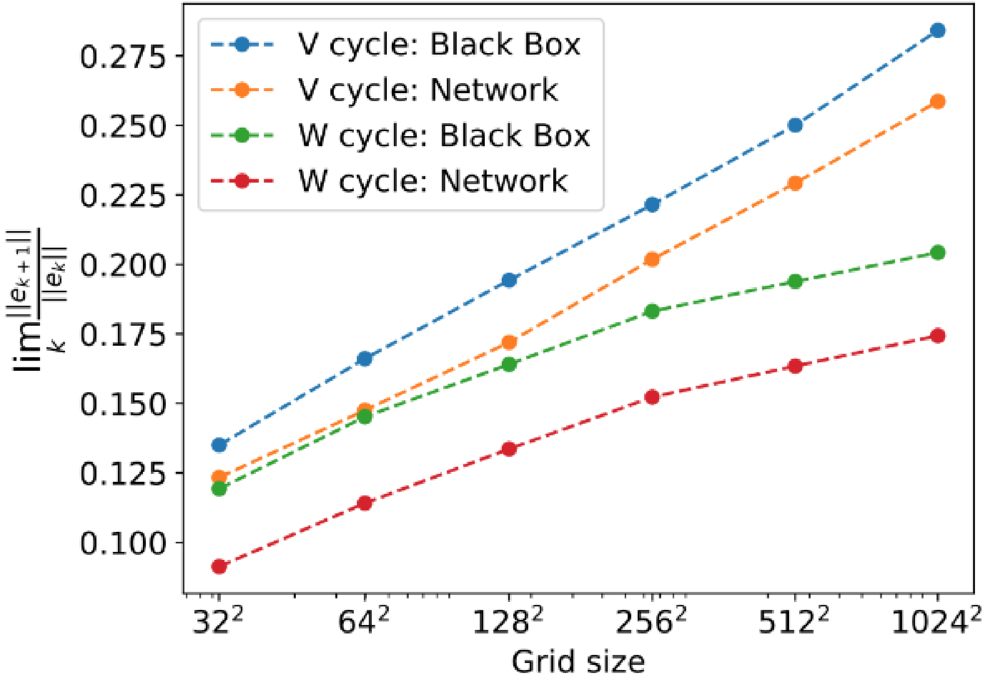
- $\rho\left(M\left(A, P_\theta(A)\right)\right)$ measures the convergence rate of the solver

- $P_\theta(A)$ is a NN mapping PDEs (discretization matrices) to multigrid solvers (prolongation operators)

- $A\sim D$ is a distribution over PDEs
(for example, a distribution over g in $-\nabla \cdot (g\nabla u) = f$)

Some results

Grid size	V cycle	W cycle
32x32	83%	100%
64x64	92%	100%
128x128	91%	100%
256x256	84%	99%
512x512	81%	99%
1024x1024	83%	98%



If interested, come check out our poster @ Pacific Ballroom #249