Emerging Convolutions *for Generative Normalizing Flows*

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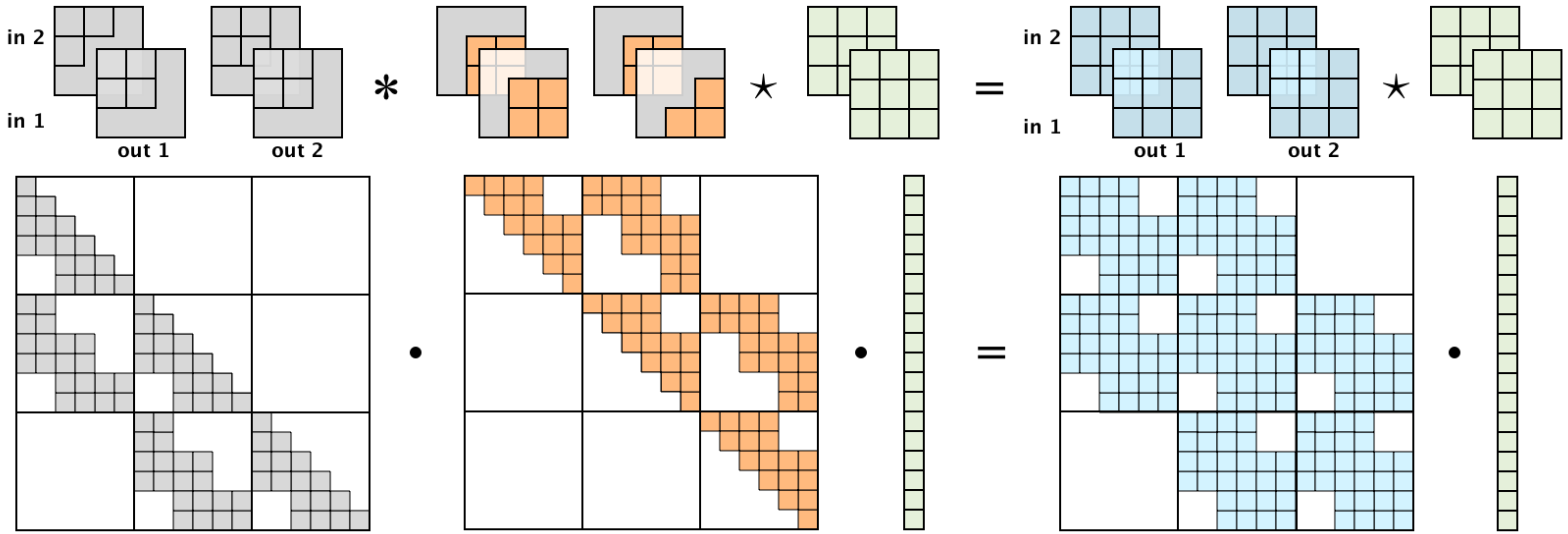
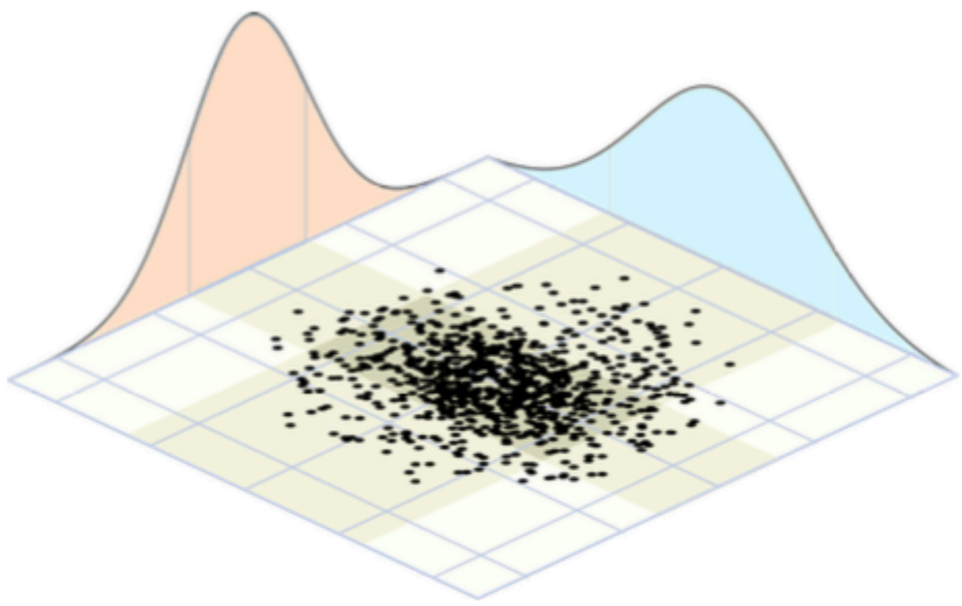


Poster: Pacific Ballroom #8

Invertible functions

$$p_X(x) = p_Z(z)\left|\frac{dz}{dx}\right| \; ; \; z = f(x)$$



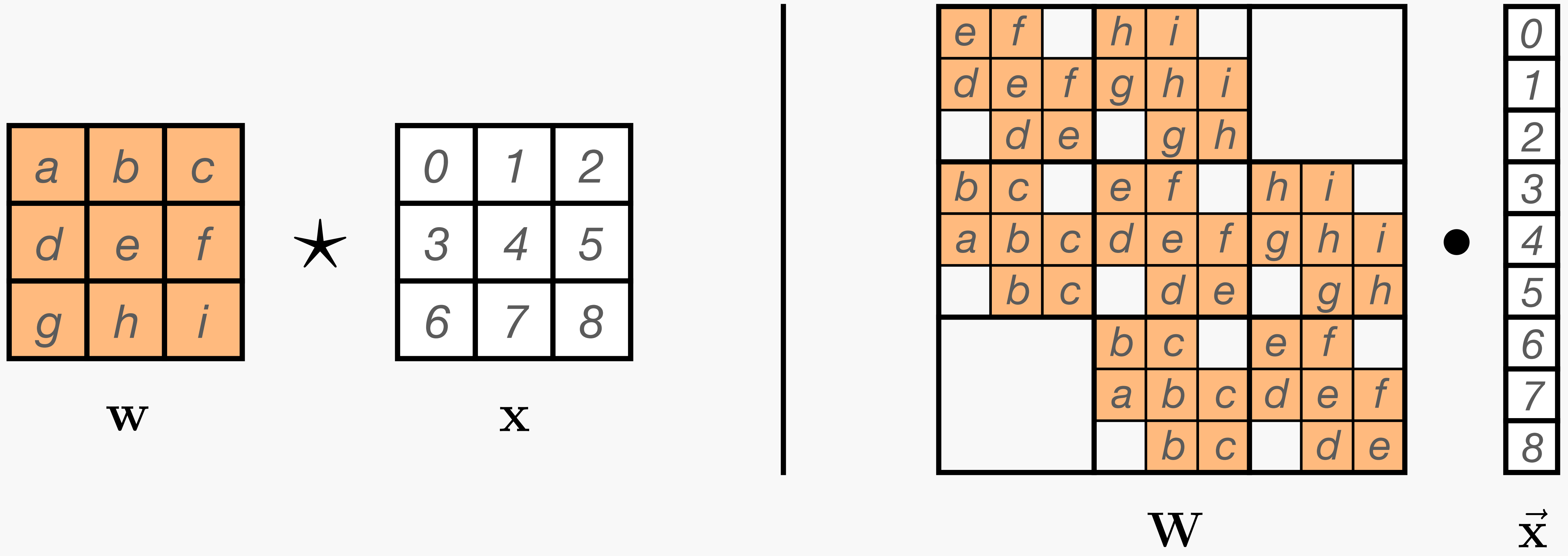
$= f($  $)$

- The change of variable formula *holds*
- Admits exact log-likelihood optimization (opposed to VAEs, GANs)
- Fast sampling (opposed to PixelCNNs)

Background

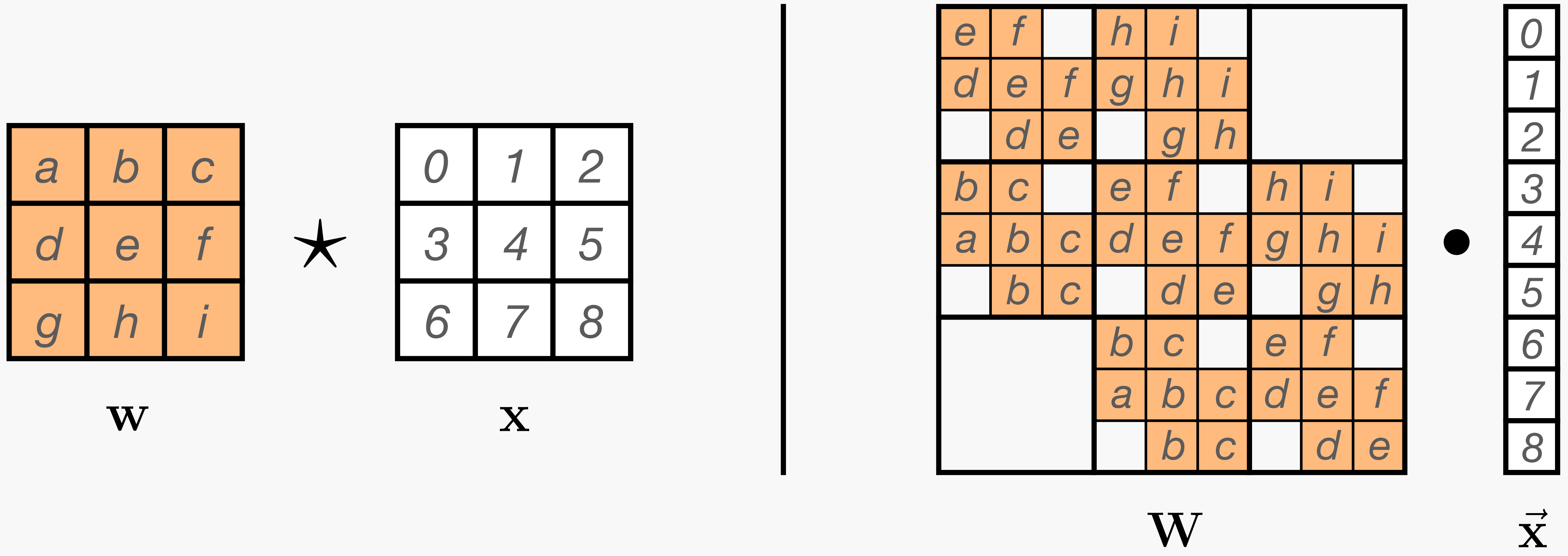
Convolutions

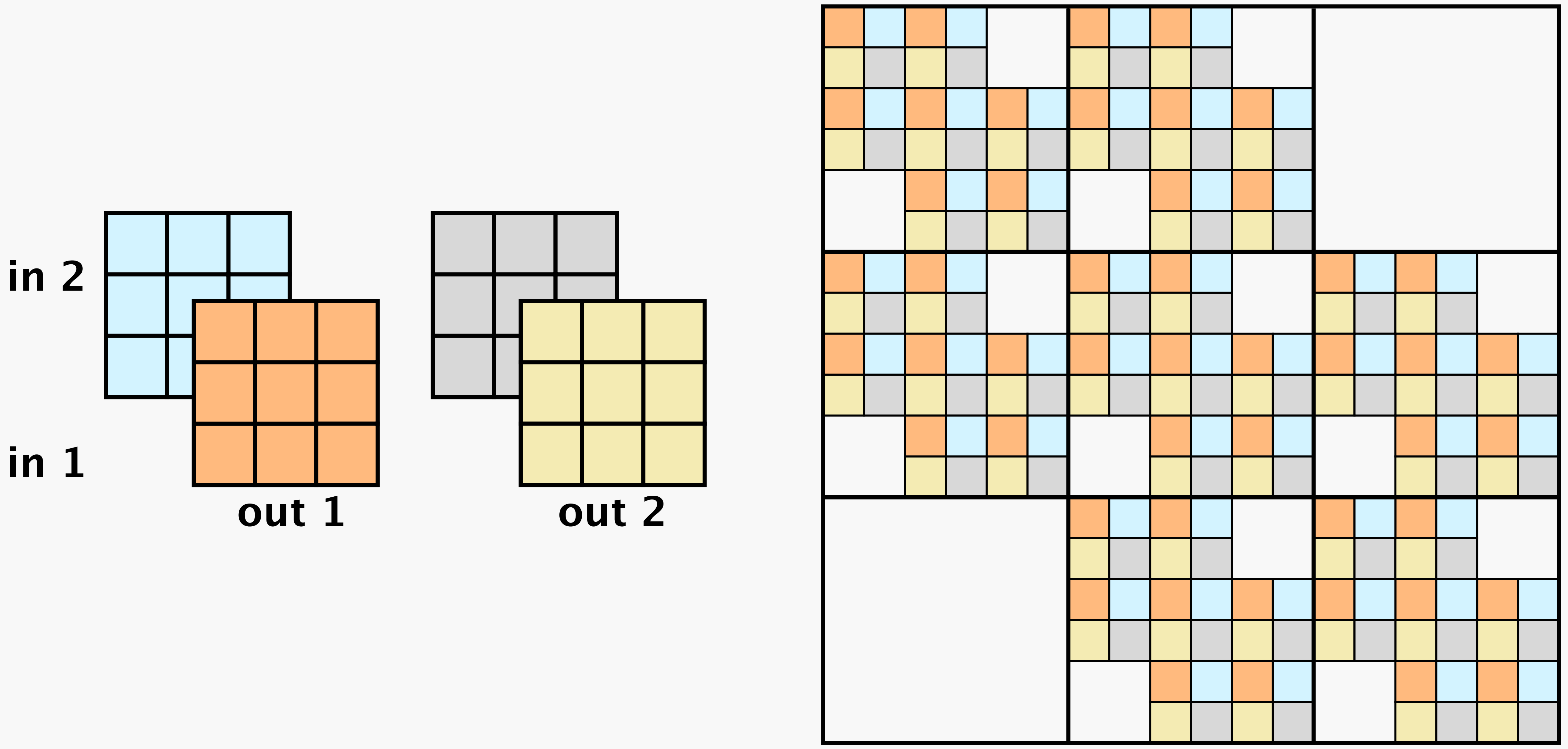
Background: Convolution as matrix multiplication



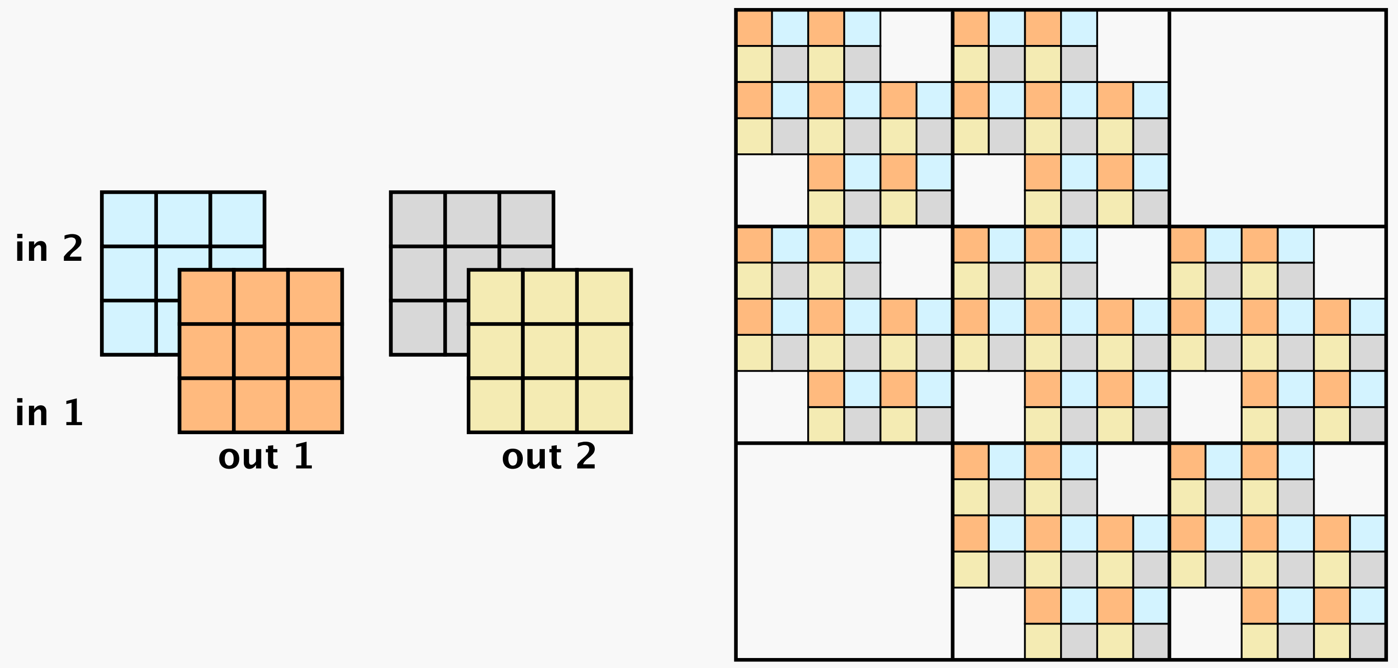
- Let **w** be a kernel, and **x** a feature map
- A convolution is equivalent to a matrix multiplication

Convolution as matrix multiplication



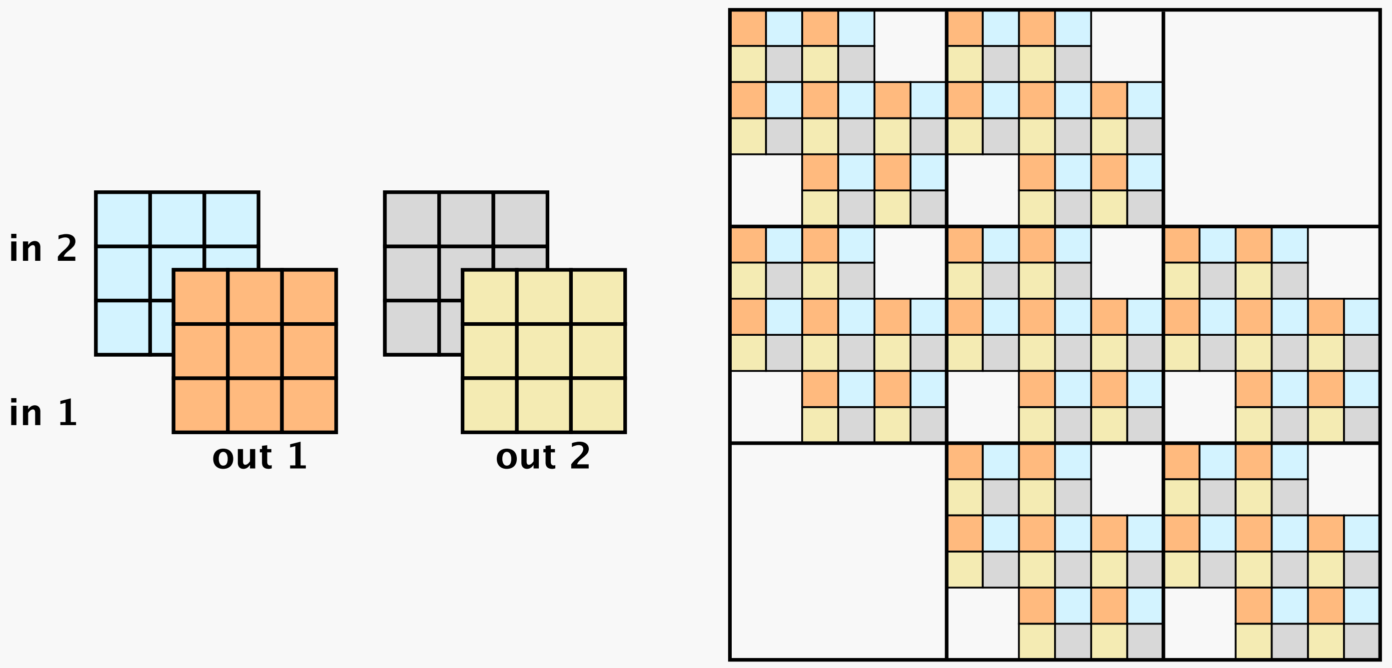


Autoregressive Convolutions

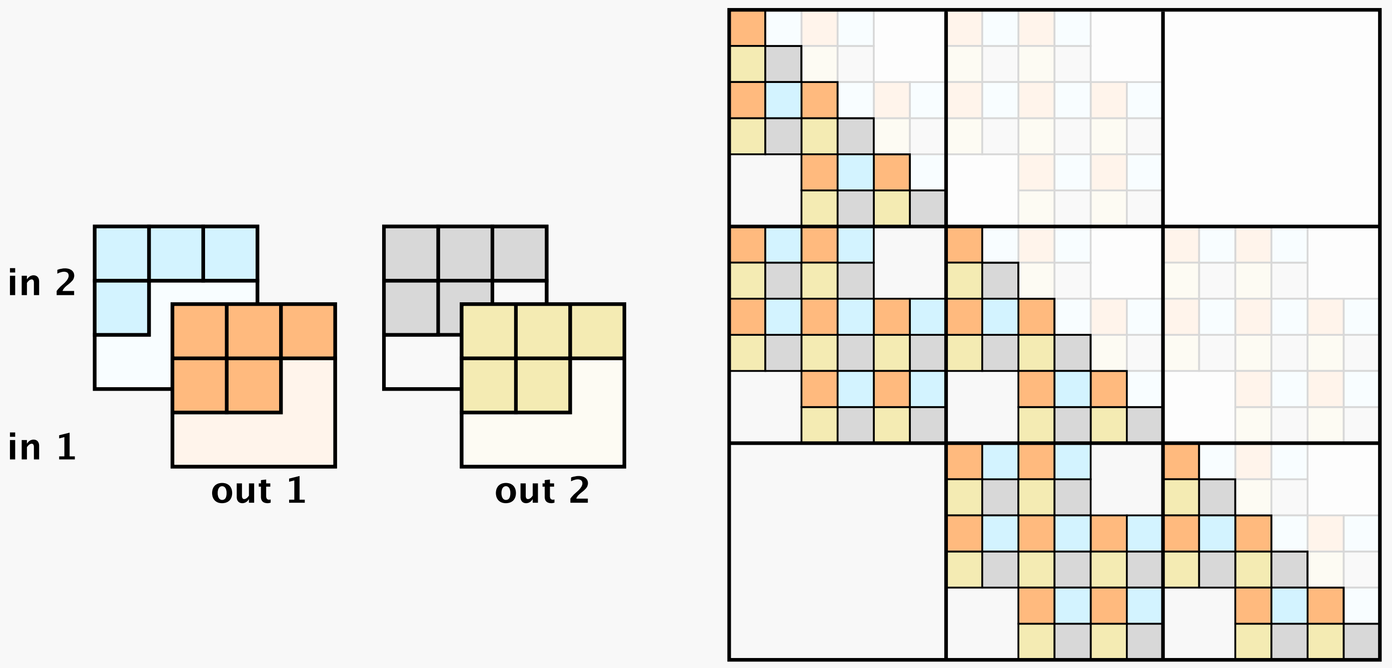


Standard convolution

Autoregressive Convolutions



Standard convolution



Autoregressive convolution

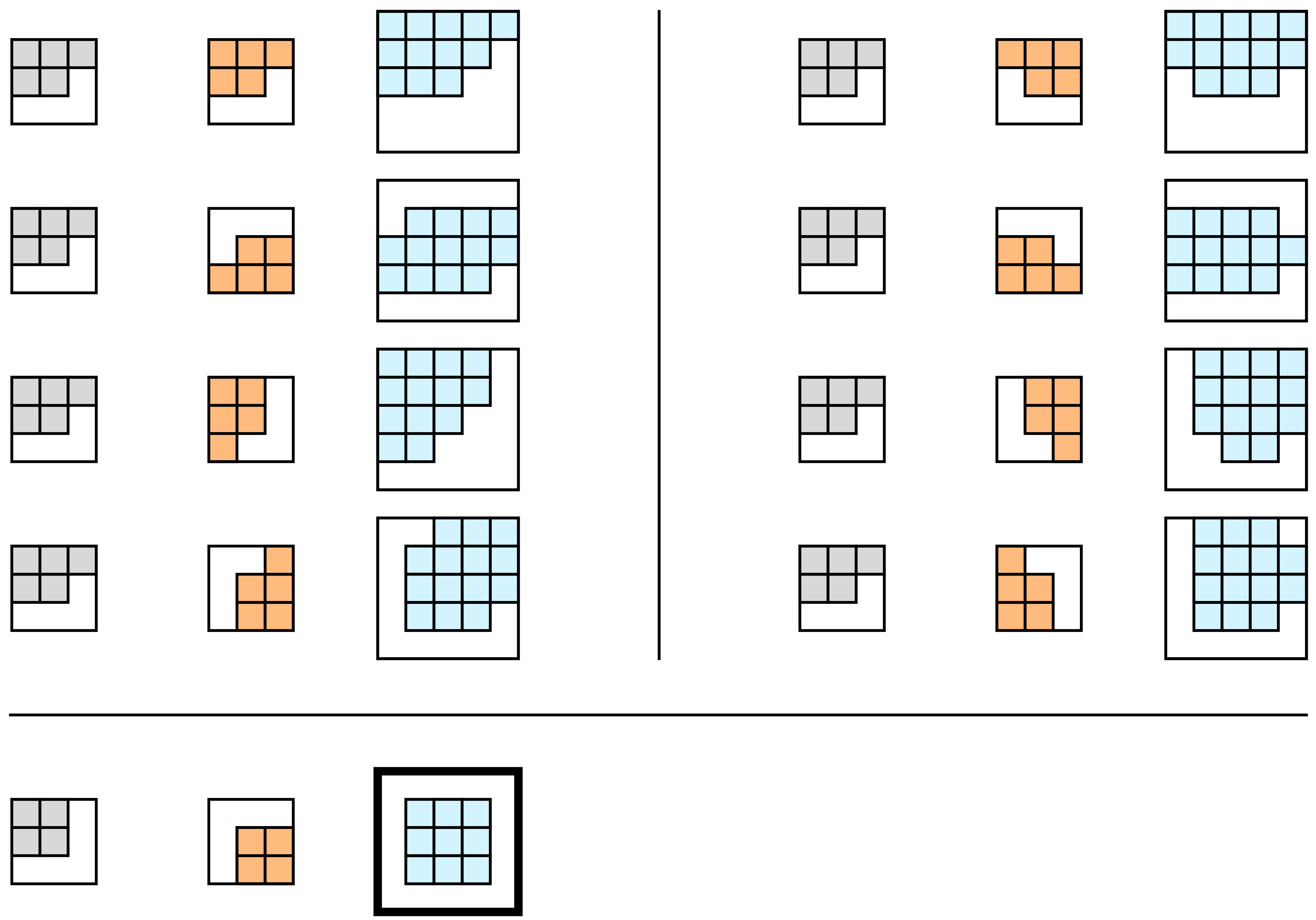
- Tractable Jacobian determinant
- Straightforward to invert

Method

Emerging convolutions

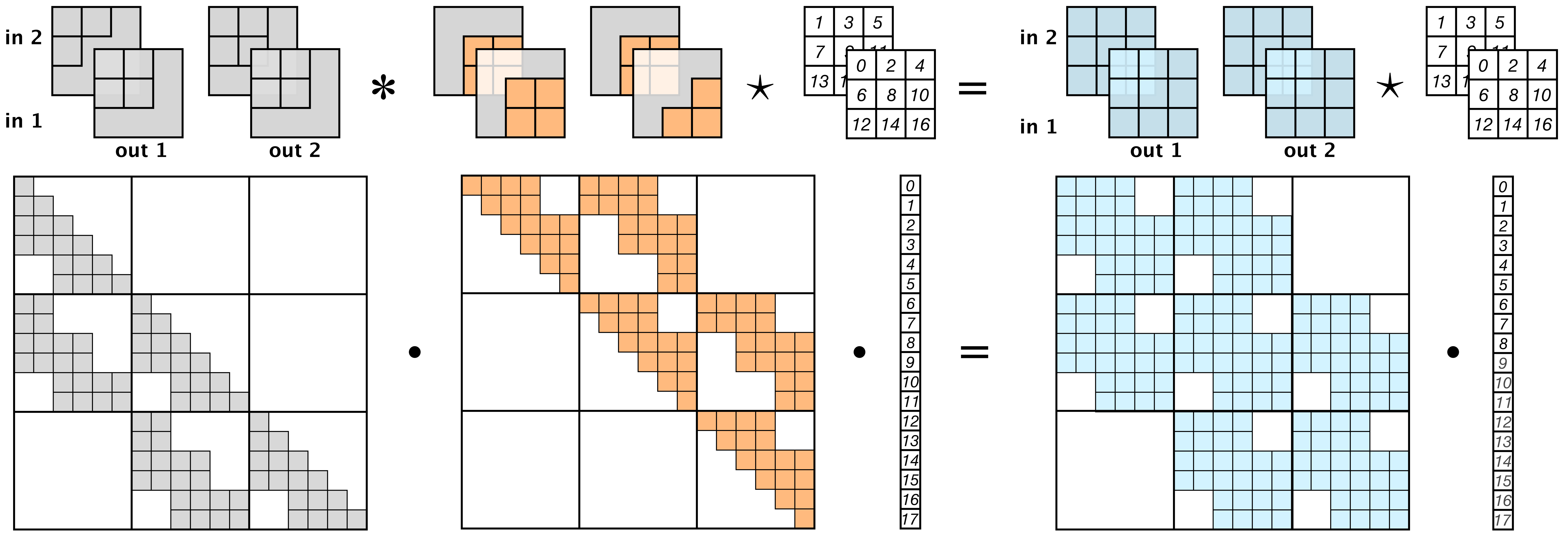
Emerging Convolutions

- Combine autoregressive convolutions
- Special case: receptive field identical to standard convolutions



Receptive fields of emerging convolutions

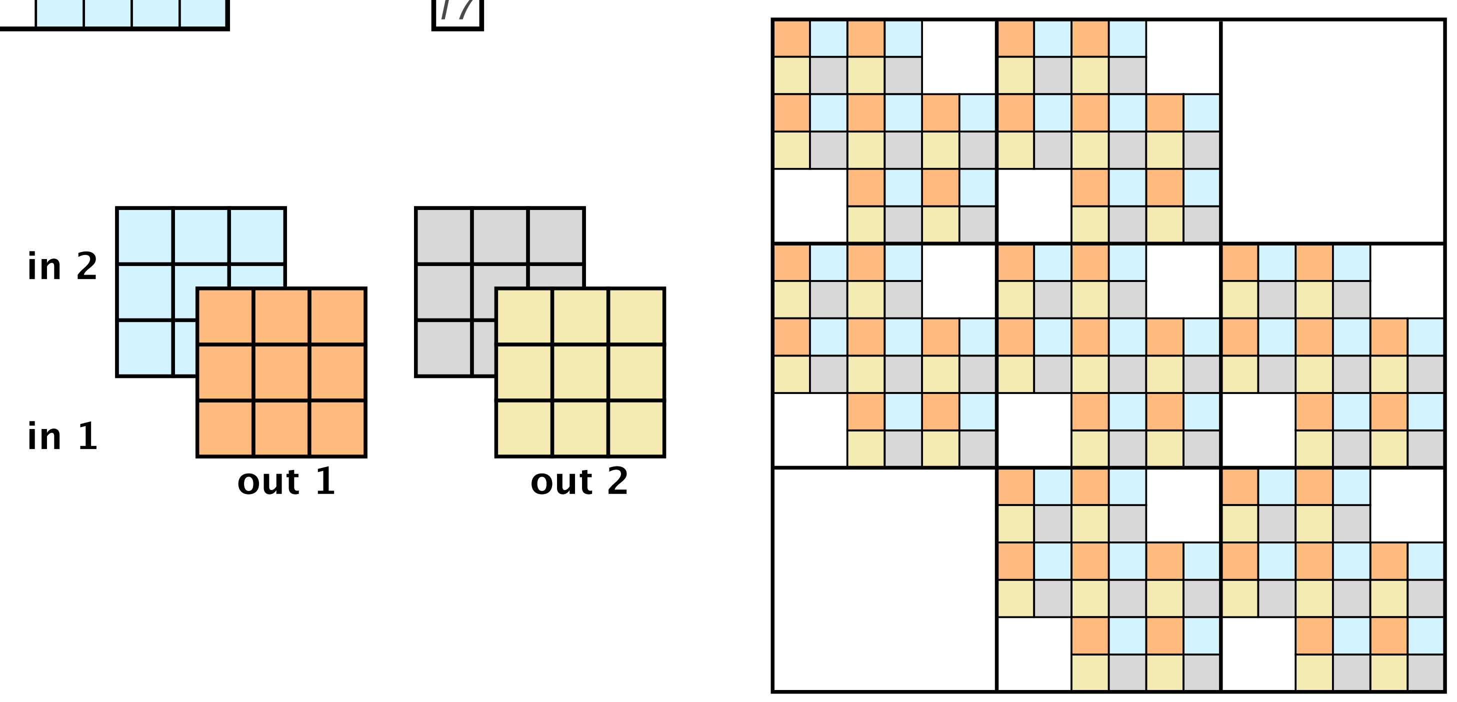
Emerging Convolutions



Emerging convolution

$$k_1 \star (k_2 \star f) = (k_1 * k_2) \star f$$

Equivalent filter



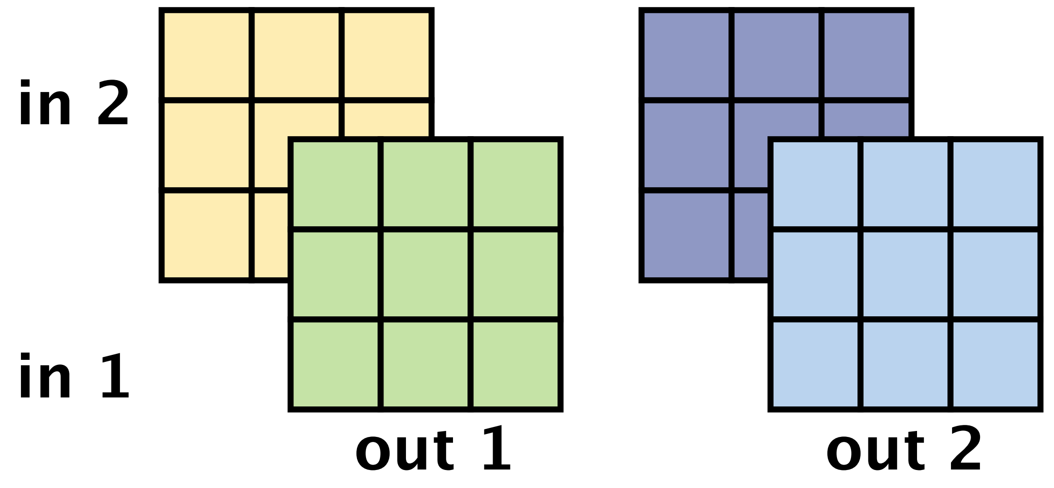
Standard convolution

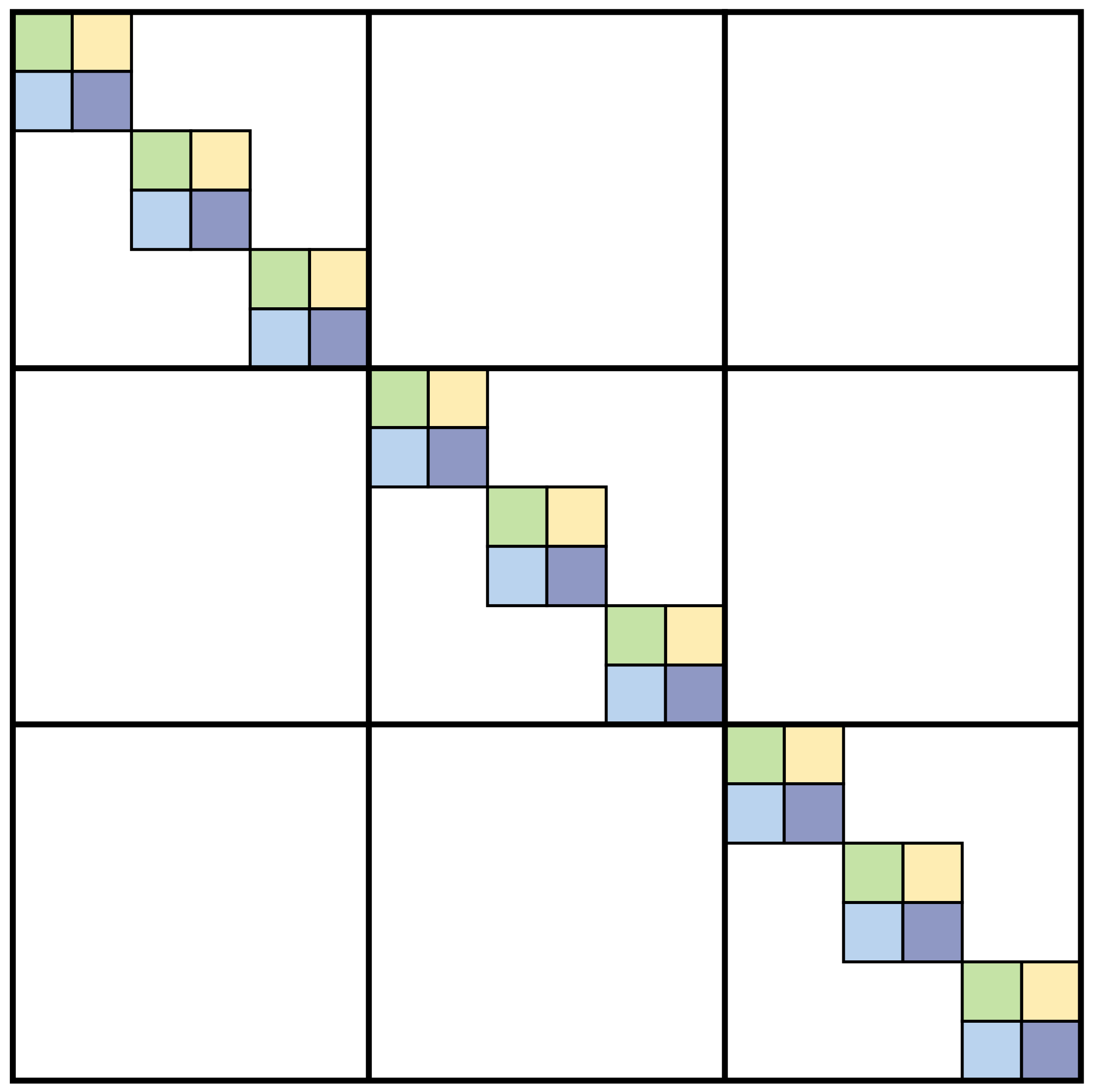
Method

Periodic convolutions

Invertible Periodic Convolutions

- Leverages the *convolution theorem*
- The determinant and inverse are computed in *frequency domain*





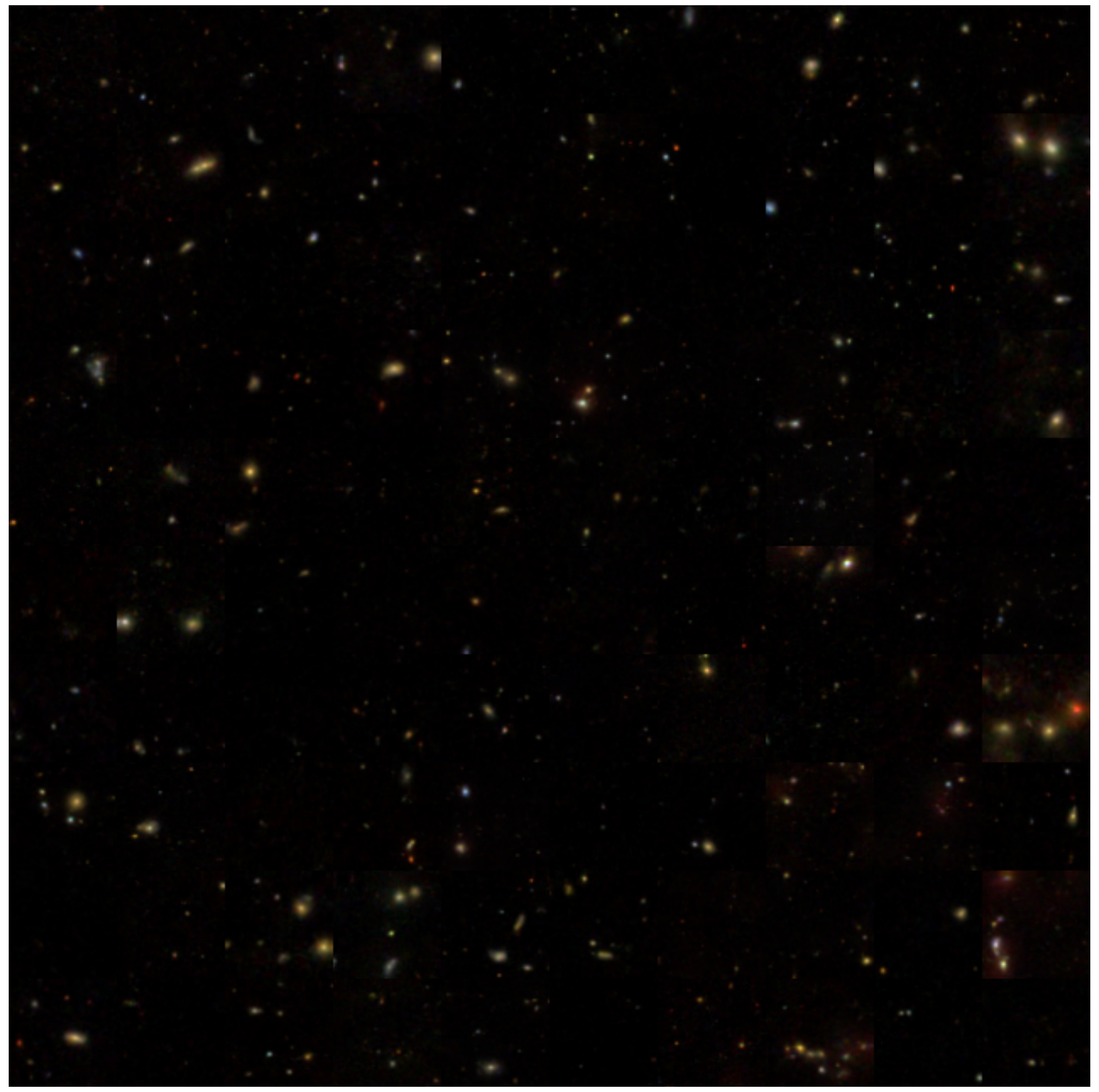
$$\hat{\mathbf{z}}_{:,uv} = \hat{\mathbf{W}}_{uv}\hat{\mathbf{x}}_{:,uv}$$

$$\hat{\mathbf{x}}_{:,uv} = \hat{\mathbf{W}}_{uv}^{-1}\hat{\mathbf{z}}_{:,uv}$$

$$\sum_{u,v} \log \left| \det \hat{\mathbf{W}}_{uv} \right|$$

Conclusion

- Emerging convolutions
- Invertible periodic convolutions
- Stable, flexible 1x1 QR convolutions
- Poster #8



	Galaxy
1×1 (Glow)	2.03 $\pm$0.026
Periodic 3×3	**1.98** $\pm$0.003
Emerging 3×3	**1.98** $\pm$0.007

	CIFAR10	ImageNet 32x32	D
1×1 (Glow)	3.46 $\pm$0.005	4.18 $\pm$0.003	8
Emerging	**3.43** $\pm$0.004	**4.16** $\pm$0.004	8
1×1 (Glow)	3.56 $\pm$0.008	4.28 $\pm$0.008	4
Emerging	**3.51** $\pm$0.001	**4.25** $\pm$0.002	4