Bayesian Joint Spike-and-Slab Graphical Lasso

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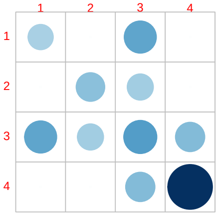
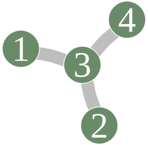
Joint work with Tyler H. McCormick (UW) and Samuel J. Clark (OSU)

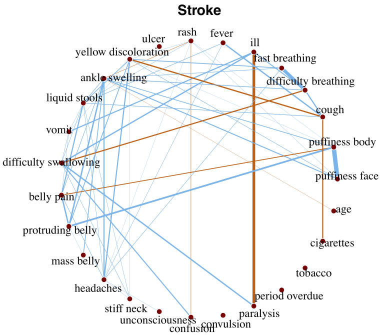
$$p(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\Omega}) = \text{Normal}(\boldsymbol{\mu}, \boldsymbol{\Omega}^{-1})$$

$$\boldsymbol{\Omega} = \begin{pmatrix} \omega_{11} & 0 & \omega_{13} & 0 \\ & \omega_{22} & \omega_{23} & 0 \\ & & \omega_{33} & \omega_{34} \\ & & & \omega_{44} \end{pmatrix}$$



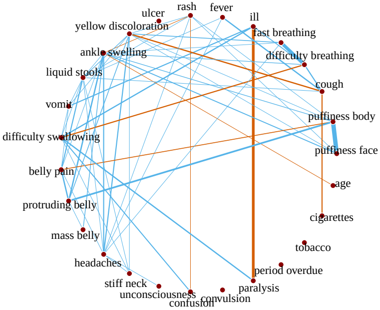


Contributions

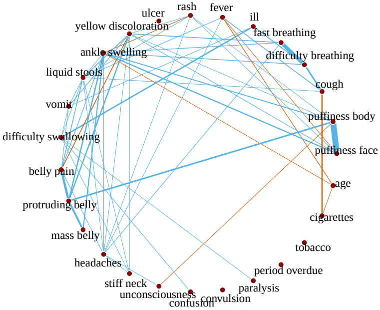


Contributions

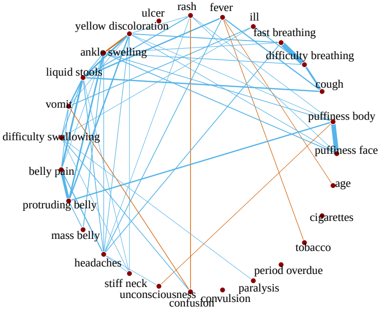
Stroke



Pneumonia



AIDS



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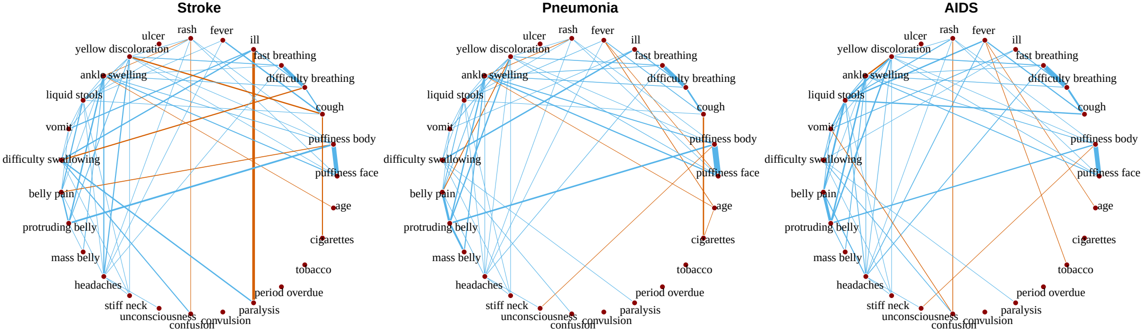

- A new graphical lasso type penalty for learning multiple related graphs.
  - Joint graphical lasso (Danaher et al., 2014)

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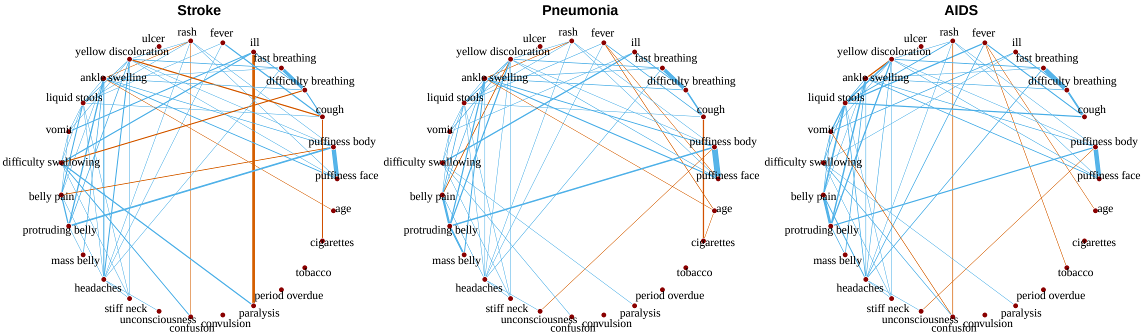
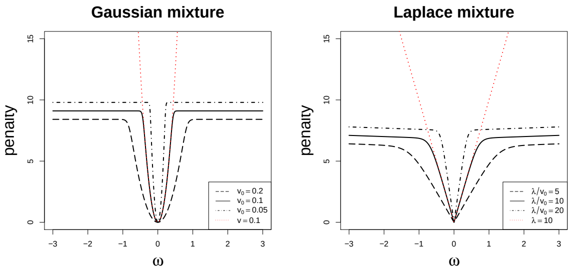


- A new graphical lasso type penalty for learning multiple related graphs.
  - Joint graphical lasso (Danaher et al., 2014)
- Reducing bias from over-shrinkage and automatic tuning parameter selection.
  - EM algorithm for graphical model (Li and McCormick, 2019)

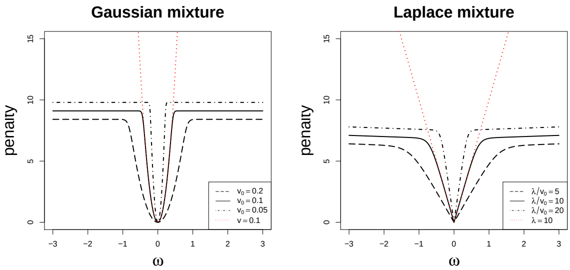
Doubly spike-and-slab joint graphical lasso priors



spike-and-slab mixture penalties

$$pen(\{\Omega\}) = \frac{\lambda_0}{2}\sum_g\sum_j |\omega_{jj}^{(g)}| + \lambda_1 \sum_g \sum_{j<k} \frac{|\omega_{jk}^{(g)}|}{v_{\delta_{jk}}} +$$

Doubly spike-and-slab joint graphical lasso priors



spike-and-slab mixture penalties

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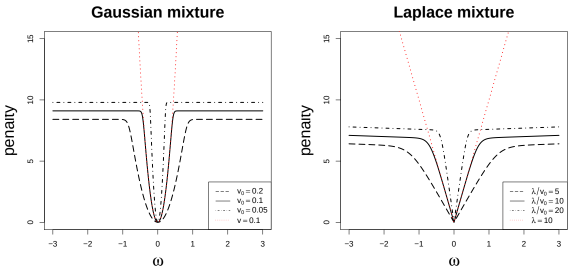
Doubly spike-and-slab joint graphical lasso priors



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Fully Bayesian characterization via scale mixture of normal priors

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E-step:

- Calculate posterior inclusion probabilities: $p^*_{\delta_{jk},\xi_{jk}}(j,k)$.
- Impute missing values.

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- Calculate posterior inclusion probabilities: $p^*_{\delta_{jk},\xi_{jk}}(j,k)$.
- Impute missing values.

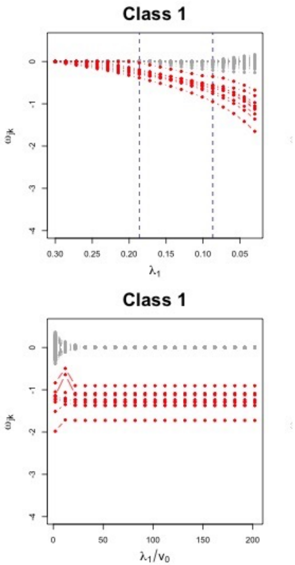
M-step:

- Solve the joint graphical lasso problem with ADMM

$$\{\widehat{\boldsymbol{\Omega}}\} = \text{argmax}_{\{\boldsymbol{\Omega}\}}\Big\{\ldots - \sum_{j<k} \lambda_1\Big(\frac{p^*_{0,0}(j,k)}{v_0} + \frac{1-p^*_{0,0}(j,k)}{v_1}\Big)\sum_g |\omega_{jk}^{(g)}| - \sum_{j<k} \lambda_2\Big(\frac{1-p^*_{1,1}(j,k)}{v_0} + \frac{p^*_{1,1}(j,k)}{v_1}\Big)\widetilde{pen}(\boldsymbol{\omega}_{jk})\Big\}$$

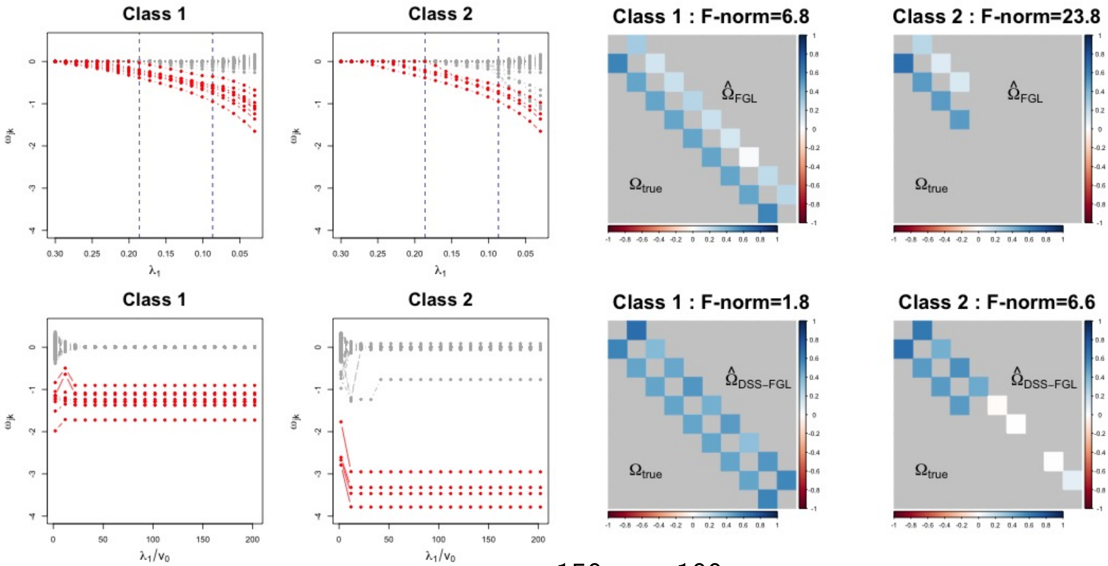
- Maximize over π_δ and π_ξ have closed form solutions

Dynamic posterior exploration



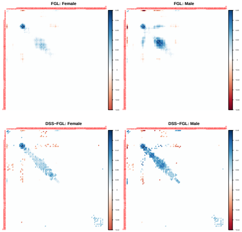
$n_1 = n_2 = 150, p = 100$

Dynamic posterior exploration

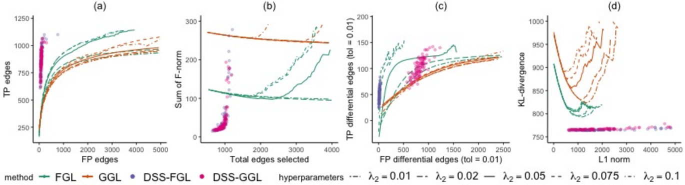


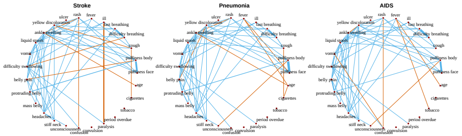
$$n_1 = n_2 = 150, p = 100$$

Verbal Autopsy and more...come to poster tonight!



Abbreviation	Questionnaire item
belly pain	For how long before death did [name] have belly pain? [days]
protruding belly	For how long before death did [name] have a protruding belly? [days]
mass belly	For how long before death did [name] have a mass in the belly [days]
headaches	For how long before death did [name] have headaches? [days]
stiff neck	For how long before death did [name] have stiff neck? [days]
unconsciousness	For how long did the period of loss of consciousness last? [days]





Get in touch? @zrichardli

References I

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Li, Z. R. and McCormick, T. H. (2019). An Expectation Conditional Maximization approach for Gaussian graphical models. *Journal of Computational and Graphical Statistics (forthcoming)*.

Ročková, V. and George, E. I. (2014). EMVS: The EM approach to Bayesian variable selection. *Journal of the American Statistical Association*, 109(506):828–846.