Revisiting Precision and Recall Definition for Generative Model Evaluation

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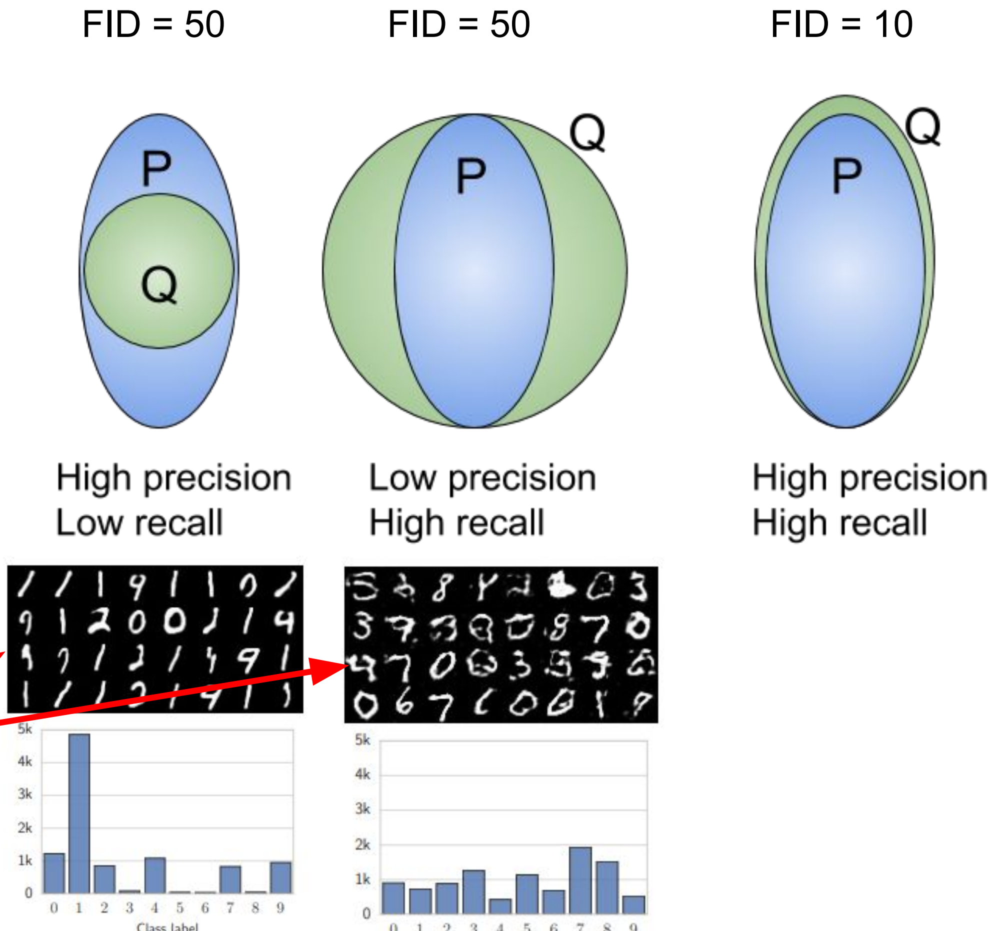
Intuitive Definition of Precision-Recall

- Precision = Probability that sample from generated distribution Q lands in the support of the target distribution P.
- Recall = Probability that sample from target distribution P lands in support of generated distribution Q.

FID is a scalar value and doesn't distinguish these two cases!



Definition of Precision Recall Curve From [Sajjadi]

- **Discrete**
- **Uses K-means to compute discrete probability density**
- Lambda defines slope of line intersecting PRD curve
- Y,X axis are max precision and recall resp.

$$\alpha(\lambda) = \sum_{\omega \in \Omega} \mathtt{min}(\lambda \mathbb{P}(\omega), \mathbb{Q}(\omega))$$

$$\beta(\lambda) = \sum_{\omega \in \Omega} \mathtt{min}(\mathbb{P}(\omega), \frac{\mathbb{Q}(\omega)}{\lambda})$$

$$\mathtt{PRD}(\mathbb{Q}, \mathbb{P}) = \{(\theta\alpha(\lambda), \theta\beta(\lambda)) \mid \lambda \in (0, \infty), \theta \in [0, 1]\}$$

$$\mathbb{Q}(\mathtt{supp}(P)) = \alpha(\infty)$$

$$\mathbb{P}(\mathtt{supp}(Q)) = \beta(0)$$

[Sajjadi et al] Assessing generative models via precision and recall, 2018, (NIPS)

Precision Recall set & curve

Definition [1]

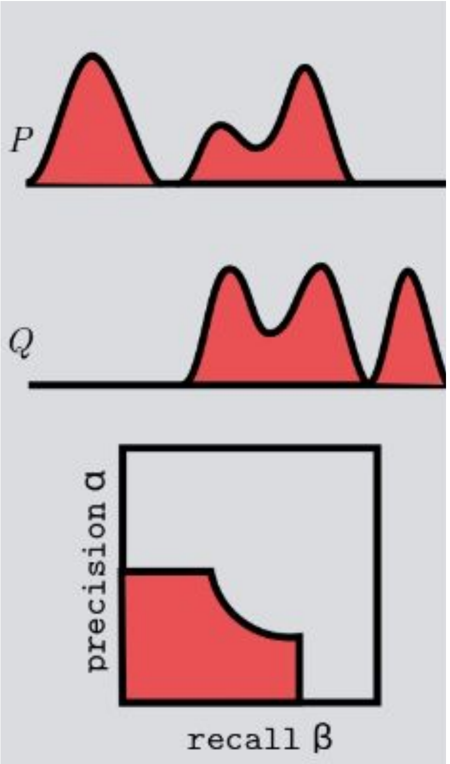
We say that $(\alpha, \beta) \in PRD(P, Q)$ iff

$$\begin{cases} \alpha \geq 0, \beta \geq 0 \\ \exists \mu \in \mathcal{M}_p(\Omega), P \geq \beta\mu, Q \geq \alpha\mu \end{cases}$$

Theorem (Pareto Front)

$(\alpha, \beta) \in PRD(P, Q)$ iff $\exists \lambda \in \overline{\mathbb{R}^+}, \alpha \leq \alpha_\lambda, \beta \leq \beta_\lambda$

with $\begin{cases} \alpha_\lambda := (\lambda P \wedge Q)(\Omega) \\ \beta_\lambda := (P \wedge \frac{Q}{\lambda})(\Omega) \end{cases}$



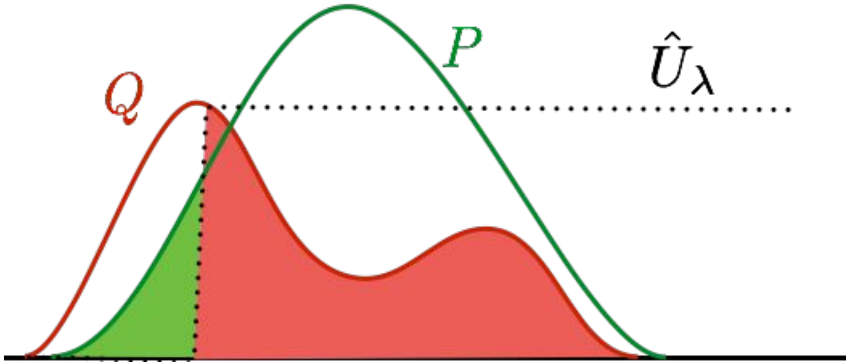
Pareto Front Reloaded

THEOREM 2

Let $Z = UX + (1-U)Y$ where $(X, Y, U) \sim P \times Q \times B_{\frac{1}{2}}$
And define a likelihood ratio classifier

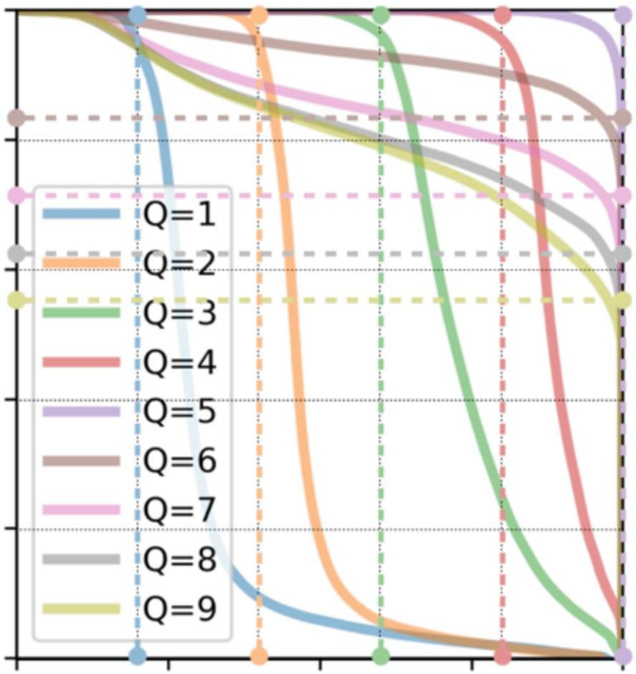
$$\hat{U}_\lambda(Z) := \mathbb{1}_{\frac{dQ}{dP}(Z) \leq \lambda}$$

Then, $$\begin{cases} \alpha_\lambda = \lambda \Pr(\hat{U}_\lambda = 0 | U = 1) + \Pr(\hat{U}_\lambda = 1 | U = 0) \\ \beta_\lambda = \Pr(\hat{U}_\lambda = 0 | U = 1) + \frac{1}{\lambda} \Pr(\hat{U}_\lambda = 1 | U = 0) \end{cases}$$

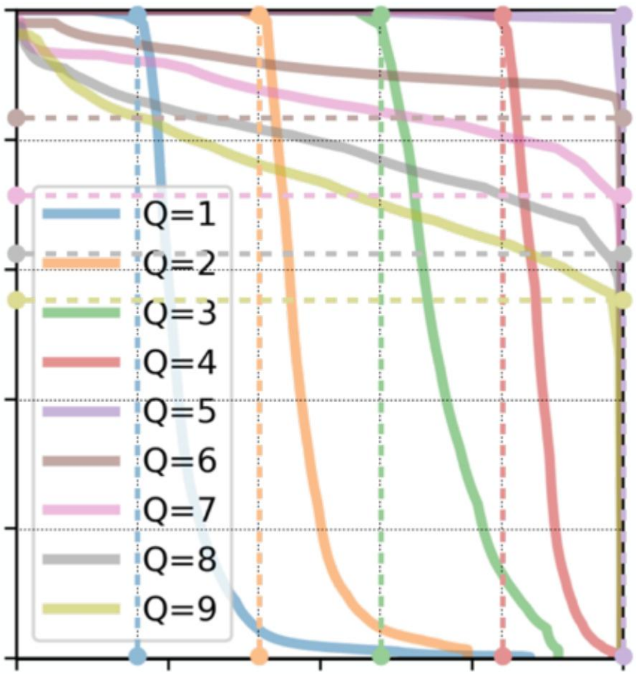


Cifar Modes

[Sajjadi et al.]

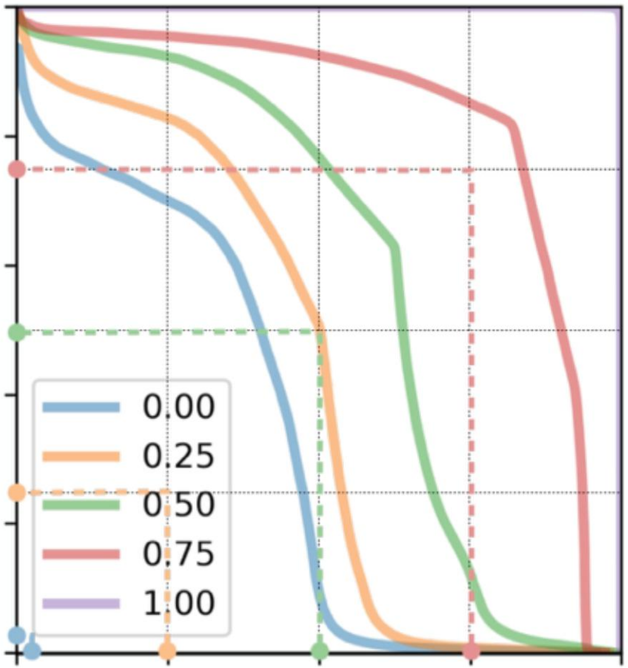


[Ours]

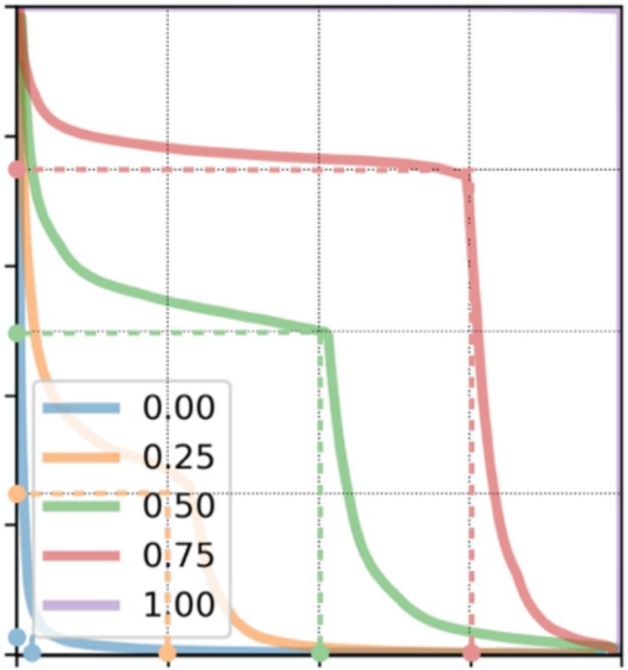


Imagenet Modes

[Sajjadi et al.]

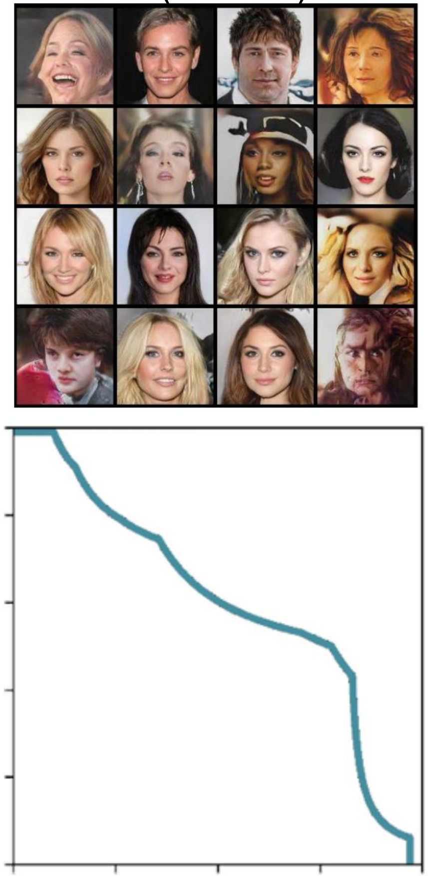


[Ours]

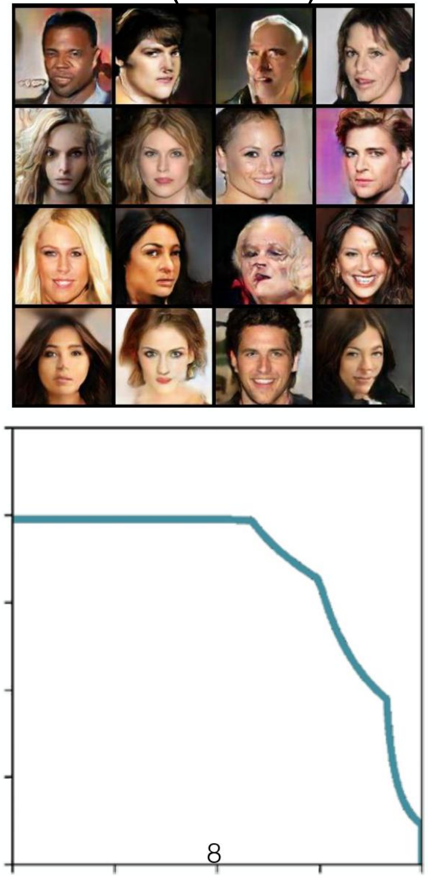


Generated images

PGGAN (FID=19)



MESCH (FID=26)



DCGAN (FID=67)