Regularization in Directable Environments with Application to Tetris

Jan Malte Lichtenberg
Özgür Şimşek

UNIVERSITY OF BATH

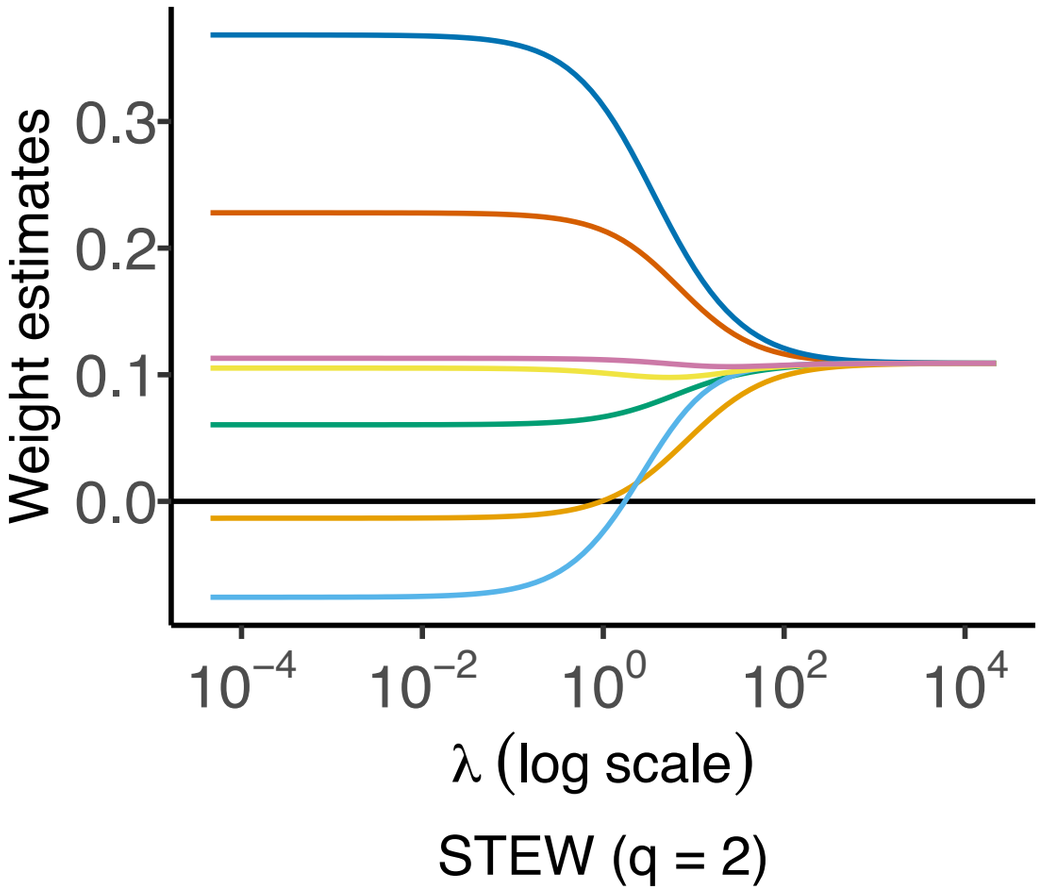
Shrinkage Toward Equal Weights (STEW)

$$\mathcal{L}^{STEW}(\boldsymbol{\beta}, \lambda) = \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\|_2^2 + \lambda \sum_{i<j} |\beta_i - \beta_j|^q$$

Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris.*

Shrinkage Toward Equal Weights (STEW)

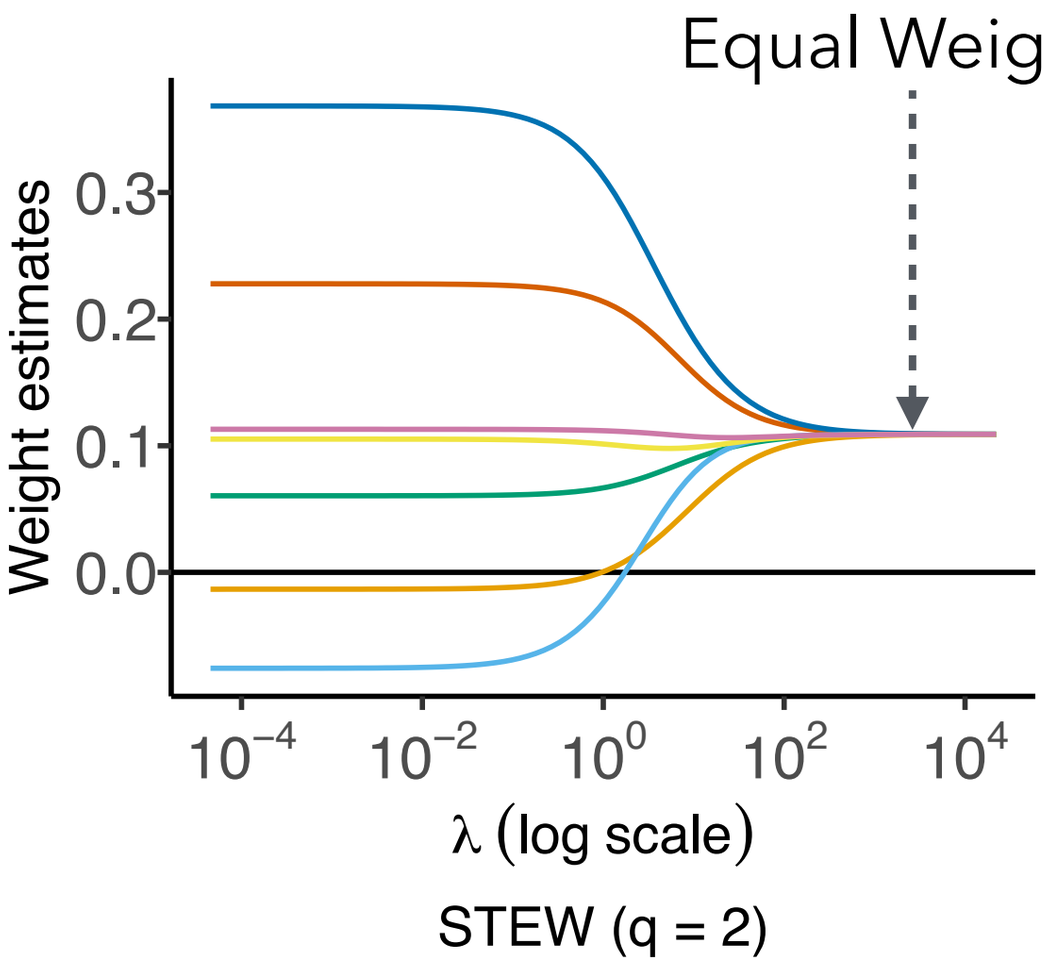
$$\mathcal{L}^{STEW}(\boldsymbol{\beta}, \lambda) = \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\|_2^2 + \lambda \sum_{i<j} |\beta_i - \beta_j|^q$$



Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris*.

Shrinkage Toward Equal Weights (STEW)

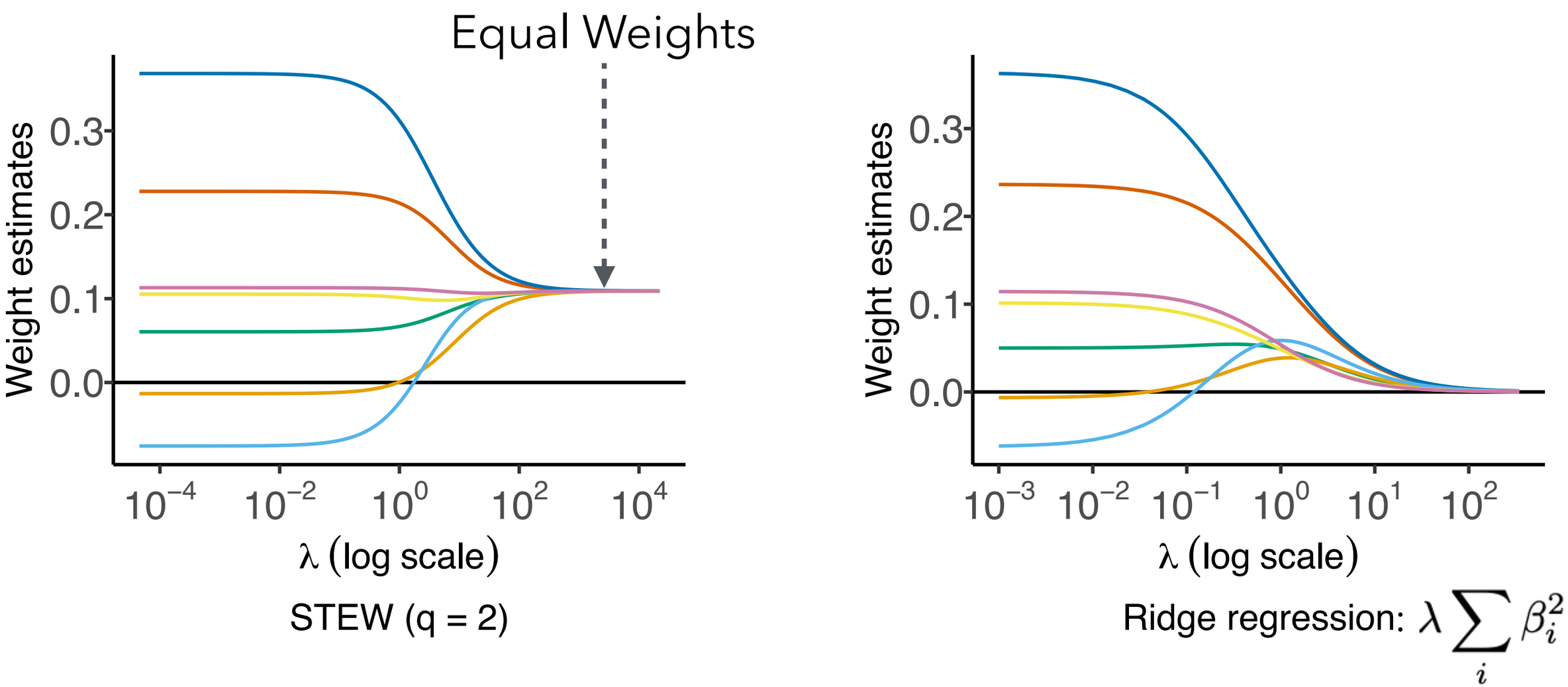
$$\mathcal{L}^{STEW}(\boldsymbol{\beta}, \lambda) = \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\|_2^2 + \lambda \sum_{i<j} |\beta_i - \beta_j|^q$$



Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris.*

Shrinkage Toward Equal Weights (STEW)

$$\mathcal{L}^{STEW}(\boldsymbol{\beta}, \lambda) = \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\|_2^2 + \lambda \sum_{i<j} |\beta_i - \beta_j|^q$$



Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris.*

Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris*.

“1 / N rule“ (DeMiguel et al., 2009)

Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris.*

$$\mathcal{L}(\boldsymbol{\beta}, \lambda) = \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\|_2^2 + \lambda \sum_{i<j} |\ |\beta_i| - |\beta_j|\ |^q$$

Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris.*

$$\mathcal{L}(\boldsymbol{\beta}, \lambda) = \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\|_2^2 + \lambda \sum_{i<j} |\ |\beta_i| - |\beta_j|\ |^q$$

Are **feature directions** known?

Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris.*

$$\mathcal{L}(\boldsymbol{\beta}, \lambda) = \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\|_2^2 + \lambda \sum_{i<j} |\ |\beta_i| - |\beta_j|\ |^q$$

Are **feature directions** known?

"Direct" all features

Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris.*

$$\mathcal{L}(\boldsymbol{\beta}, \lambda) = \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\|_2^2 + \lambda \sum_{i<j} |\ |\beta_i| - |\beta_j|\ |^q$$

Are **feature directions** known?

↓

"Direct" all features

↓

$$\mathcal{L}^{STEW}(\boldsymbol{\beta}, \lambda) = \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\|_2^2 + \lambda \sum_{i<j} |\beta_i - \beta_j|^q$$

Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris.*

$$\mathcal{L}(\boldsymbol{\beta}, \lambda) = \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\|_2^2 + \lambda \sum_{i<j} |\ |\beta_i| - |\beta_j|\ |^q$$

Are **feature directions** known?

↓

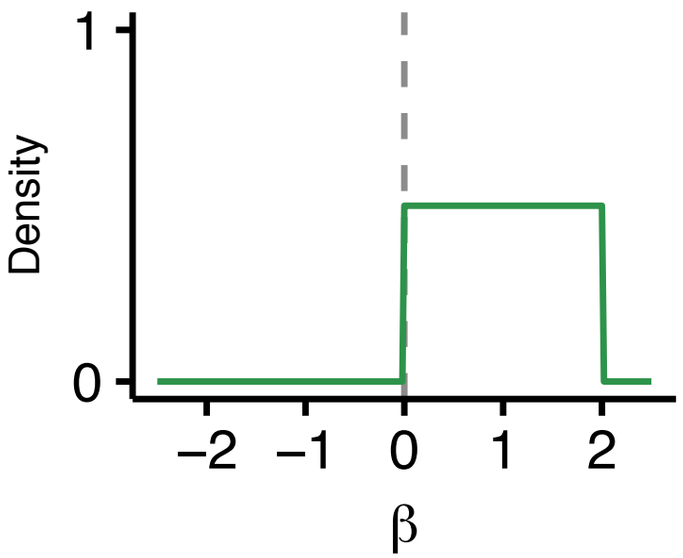
"Direct" all features

↓

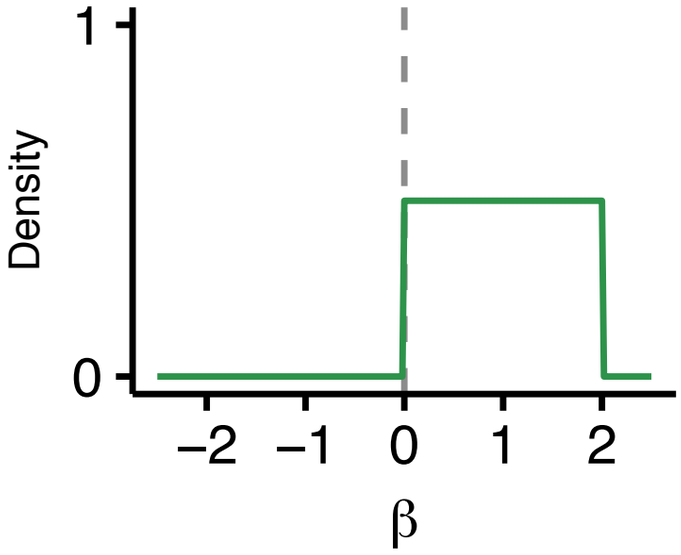
$$\mathcal{L}^{STEW}(\boldsymbol{\beta}, \lambda) = \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\|_2^2 + \lambda \sum_{i<j} |\beta_i - \beta_j|^q$$

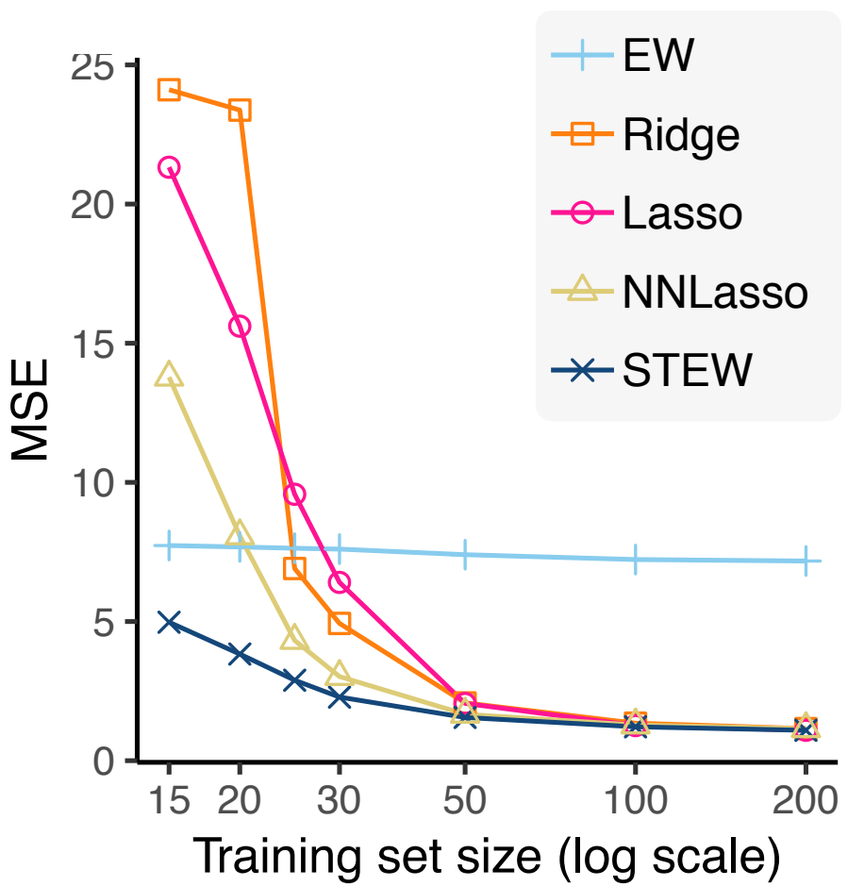
$$\text{for } q = 2,\ \underset{\boldsymbol{\beta}}{\operatorname{argmin}}\ \mathcal{L}^{STEW}(\boldsymbol{\beta}, \lambda) = (\boldsymbol{X}^T\boldsymbol{X} + \lambda \boldsymbol{D}^T\boldsymbol{D})^{-1}\boldsymbol{X}^T\boldsymbol{y}$$

Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris.*



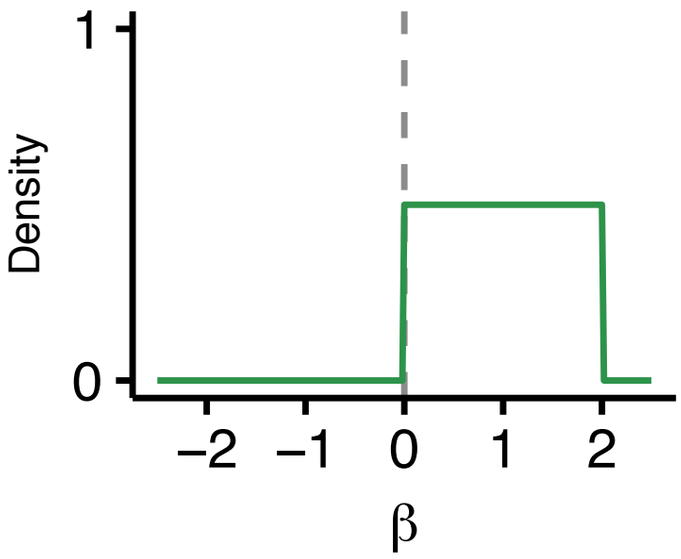
Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris.*

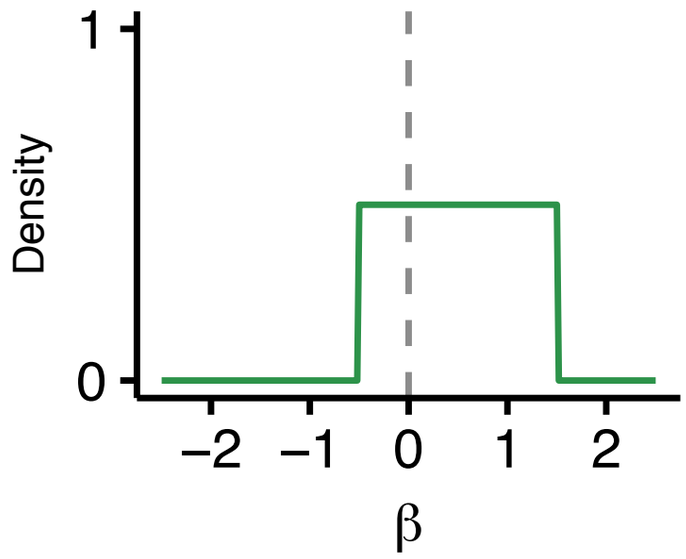


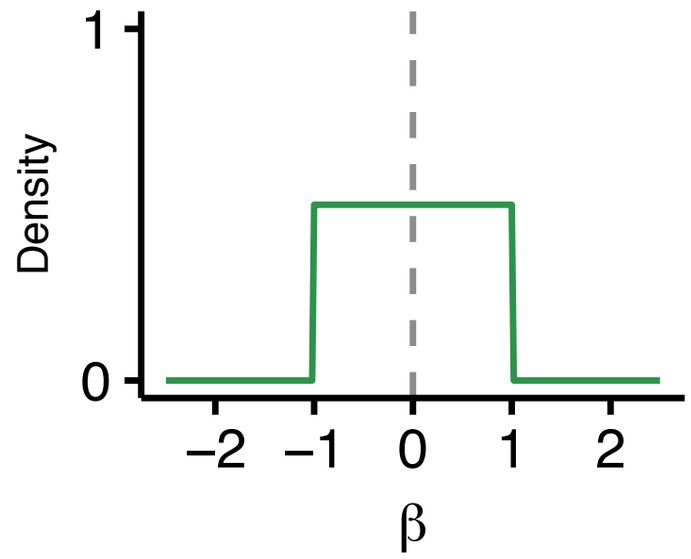


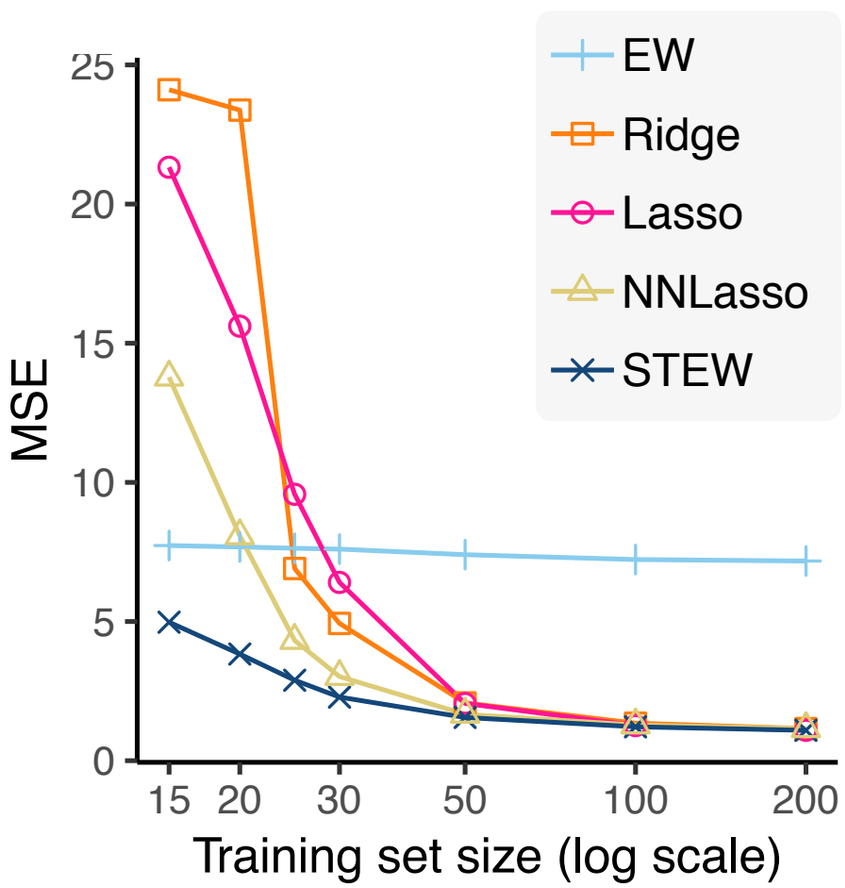
Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris.*

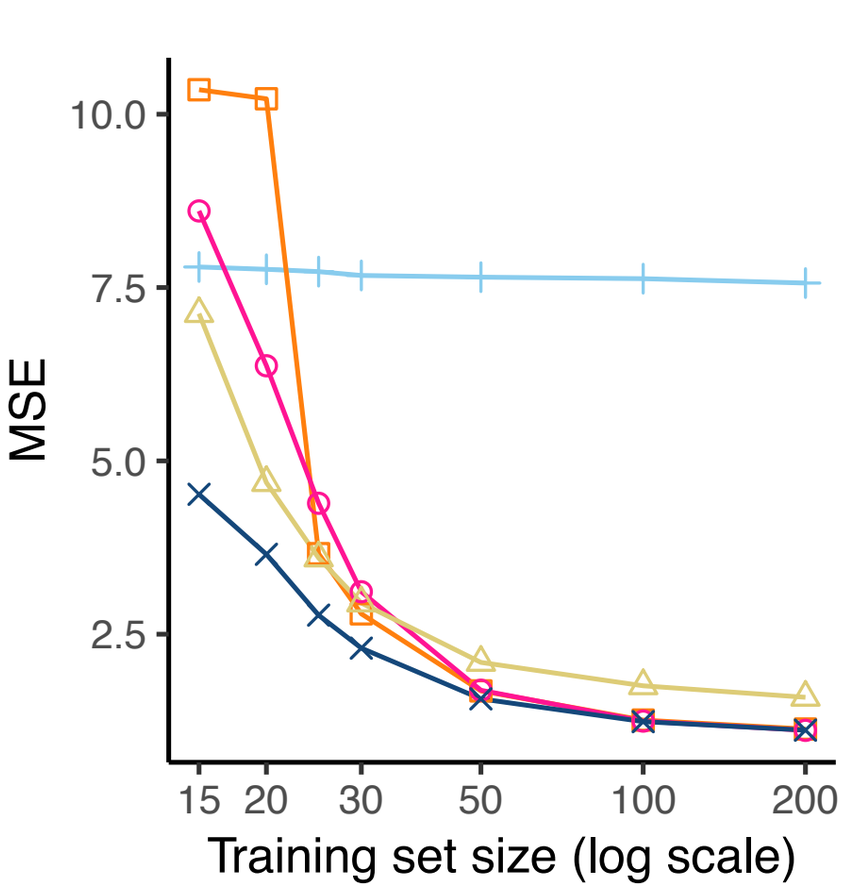
Environment less directable

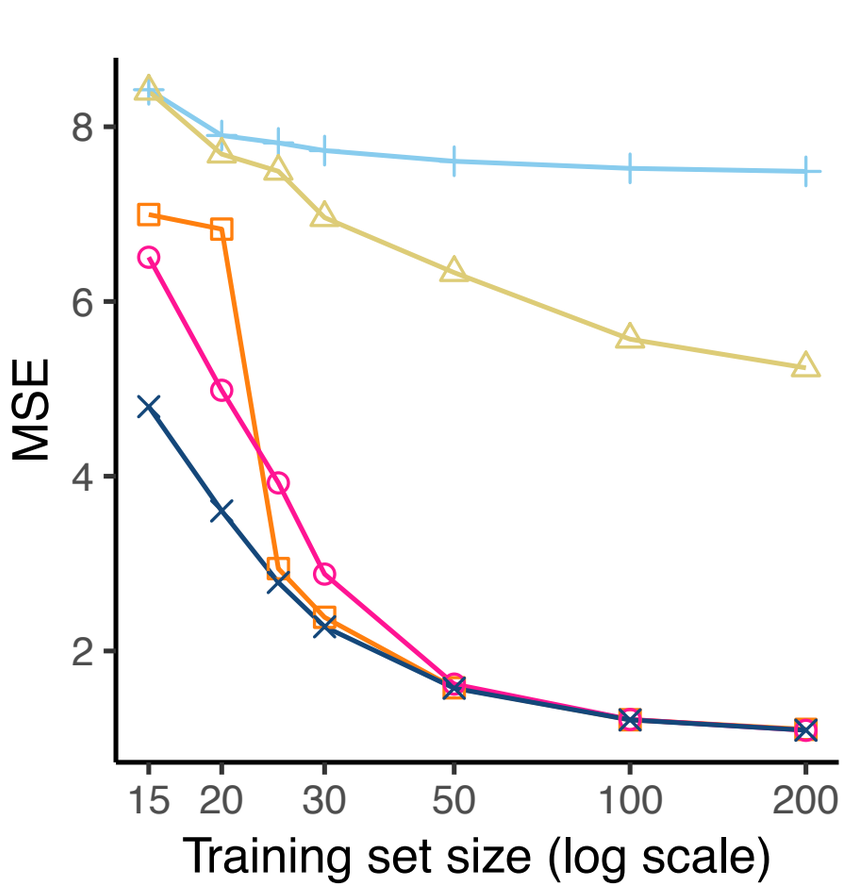




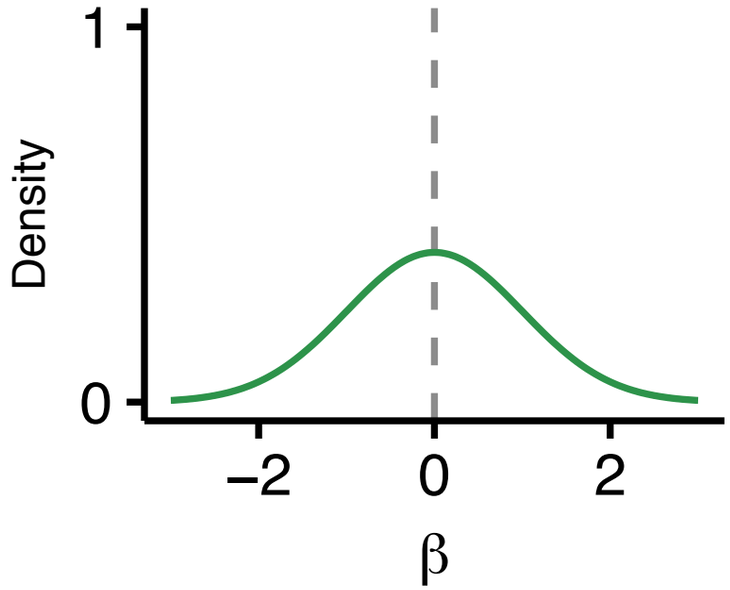


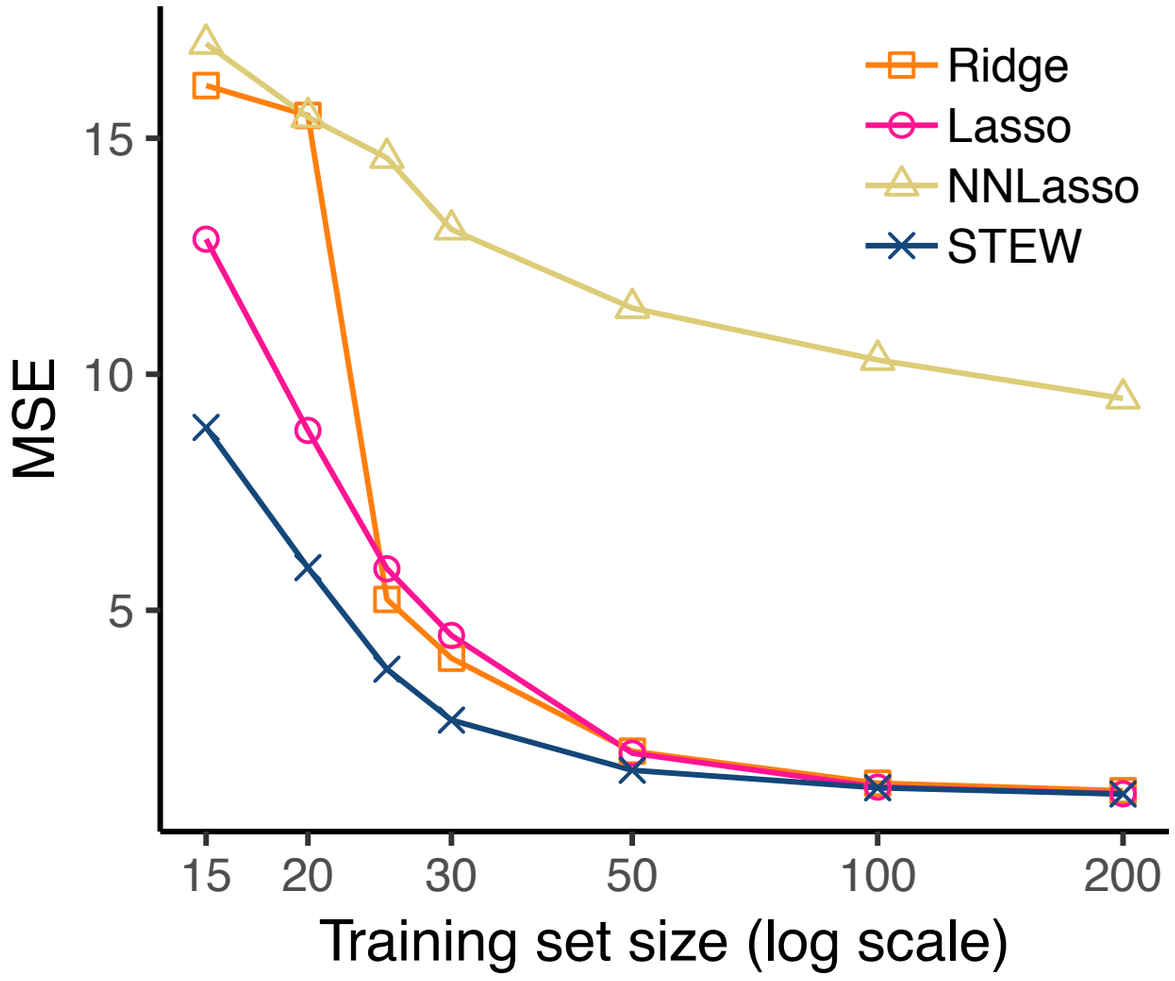




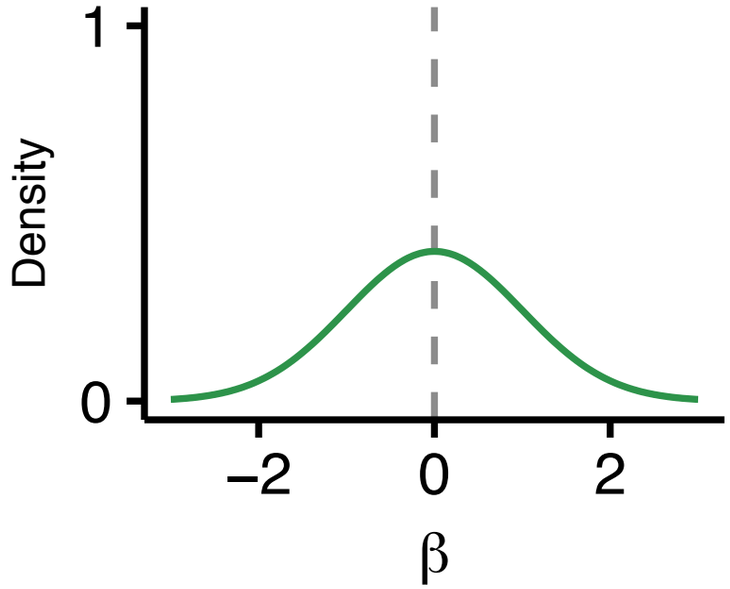


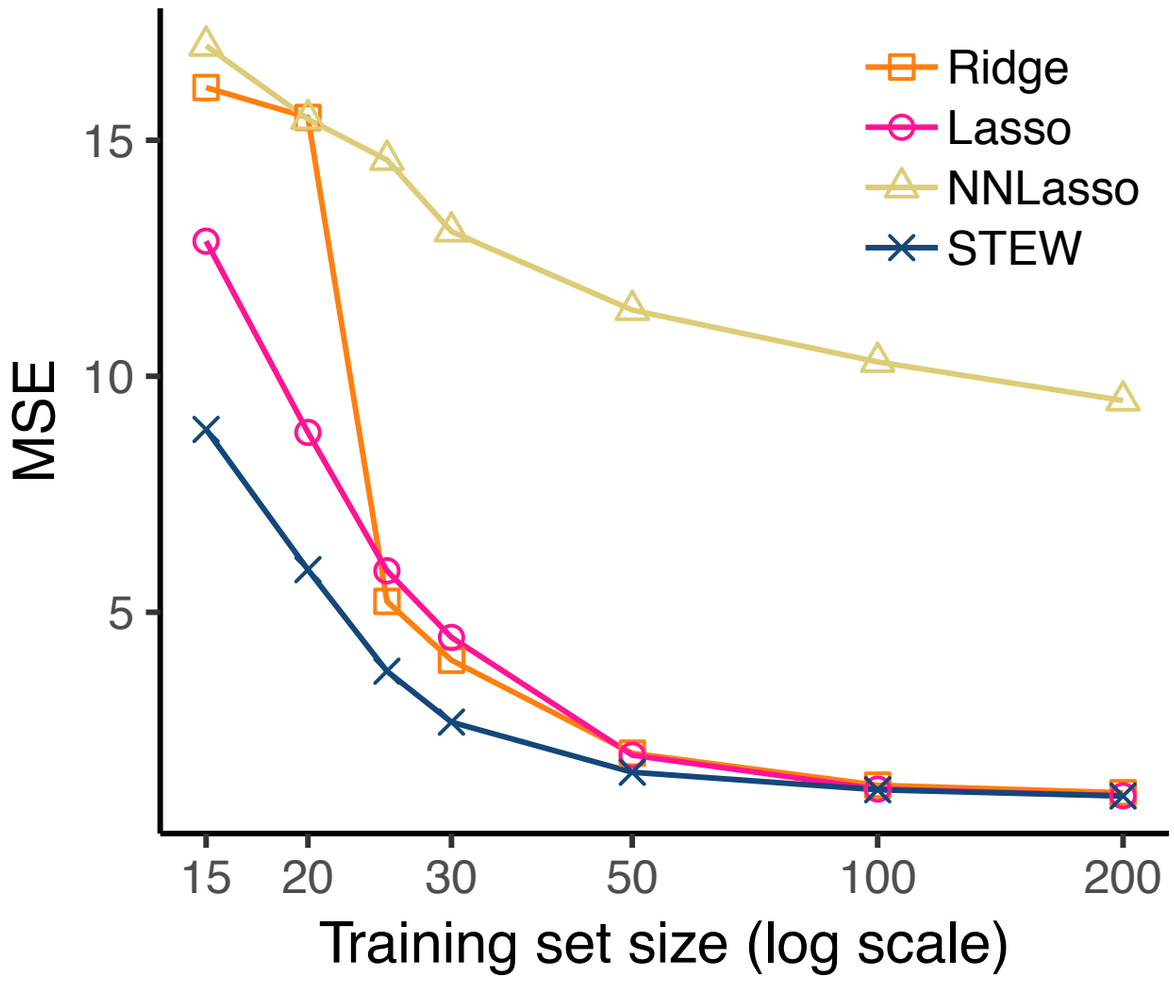
Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris.*

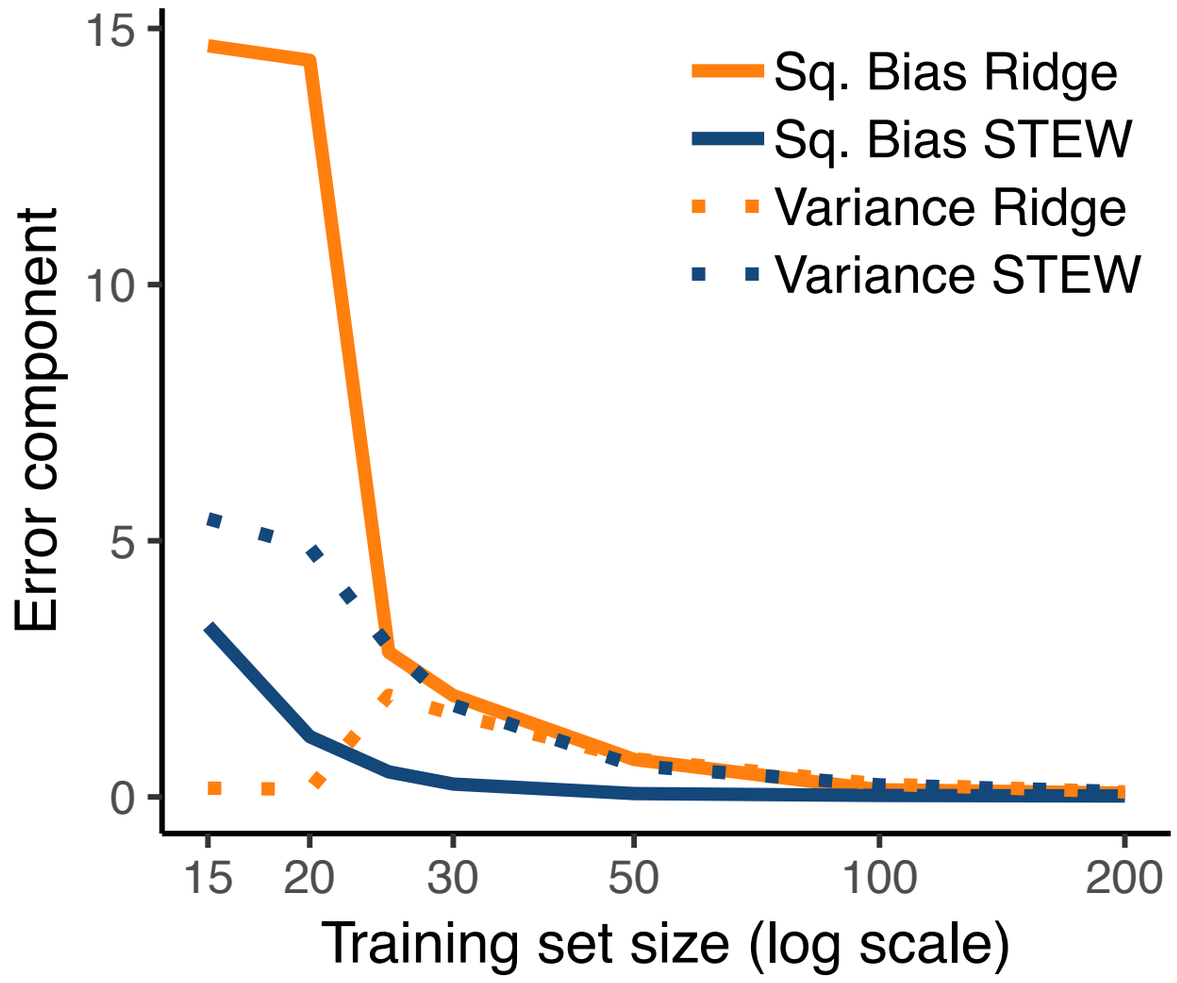




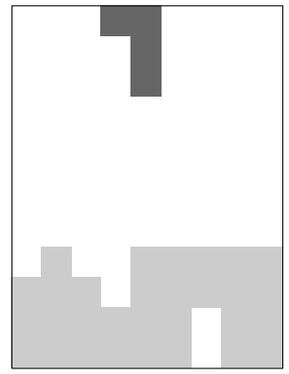
Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris.*



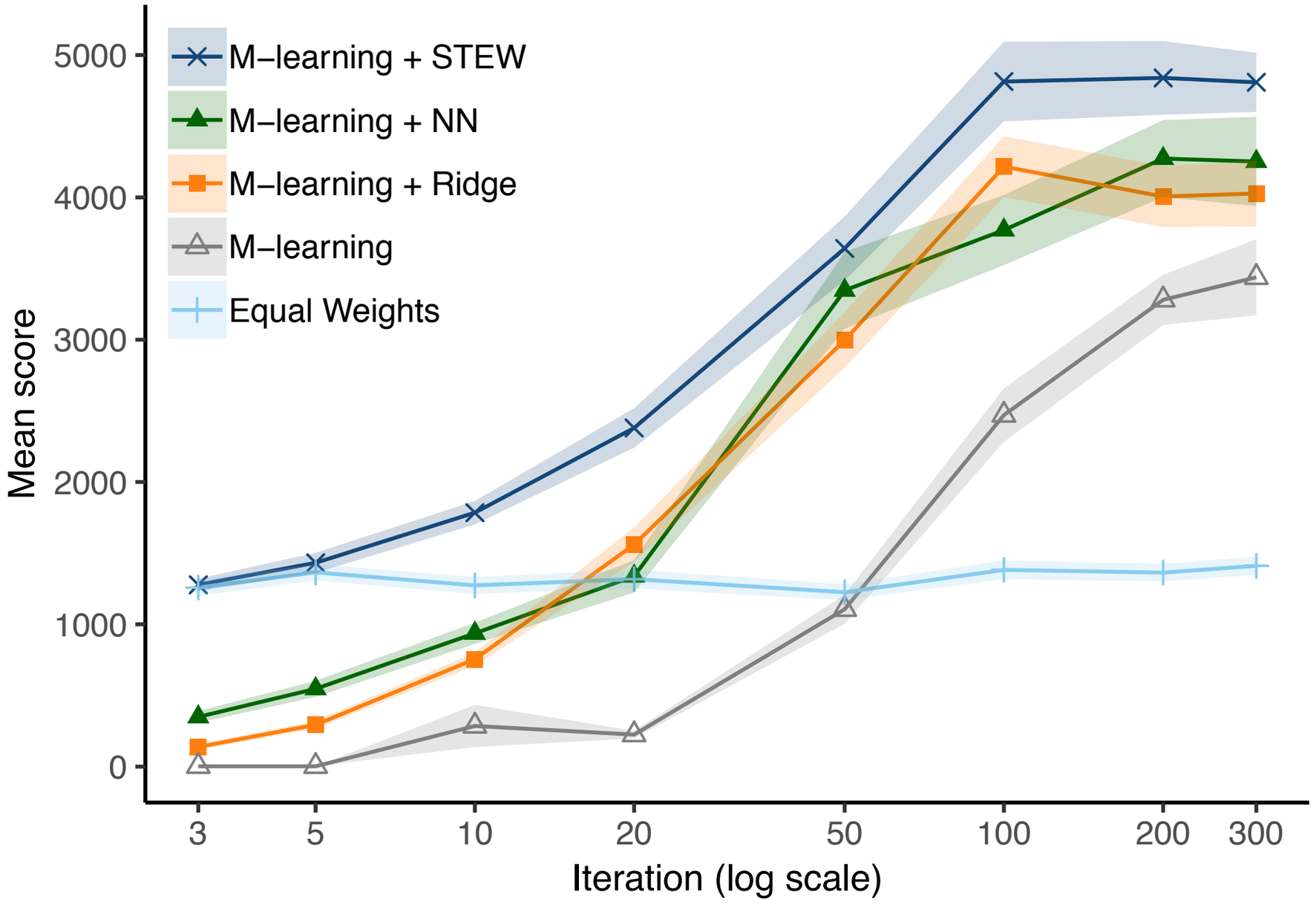




Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris.*



Tetris



Poster **#137** @ Pacific Ballroom

Lichtenberg J.M. and Şimşek Ö. *Regularization in Directable Environments with Application to Tetris.*