On the Universality of Invariant Networks

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Invariant tasks

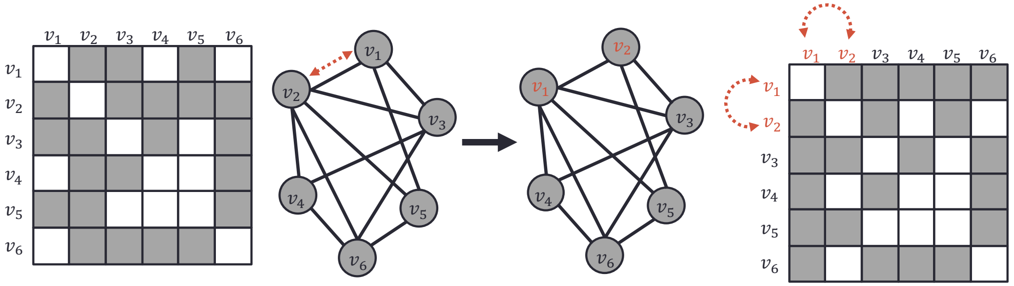
- Image classification

Car Car

Invariant tasks

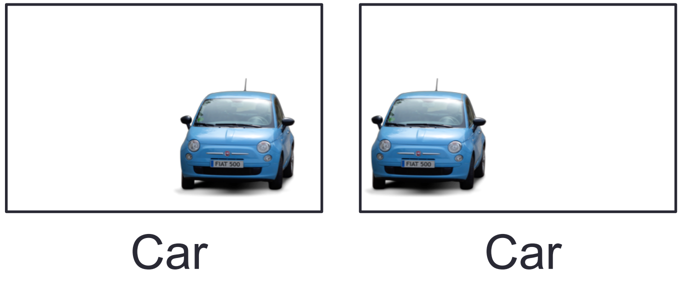
- Image classification

- Graph/ hyper-graph classification

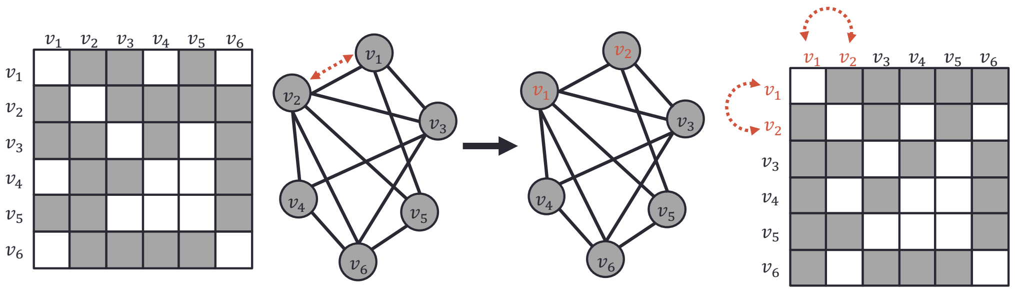


Invariant tasks

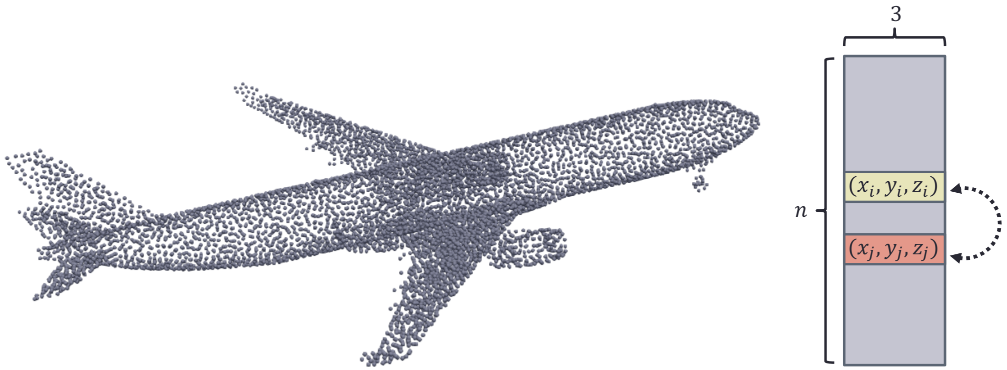
- Image classification



- Graph/ hyper-graph classification



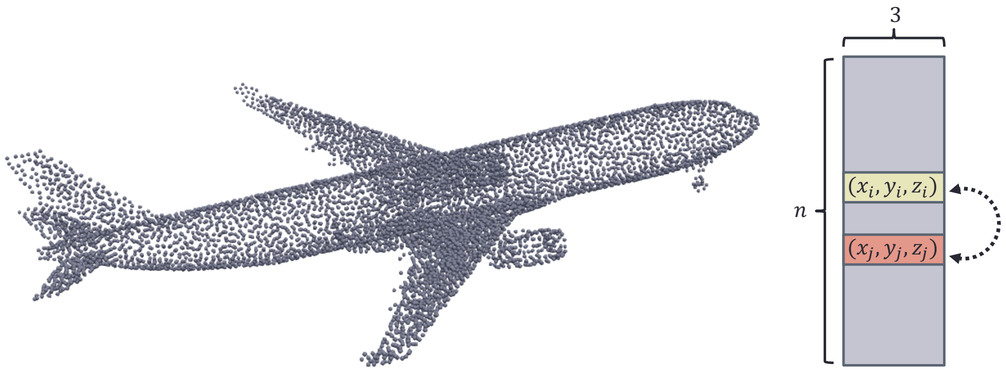
- Point-cloud / set classification



- and many more…

Goal of this paper

- Invariant neural networks are a common approach for these tasks
- This paper analyzes the expressive power of invariant models



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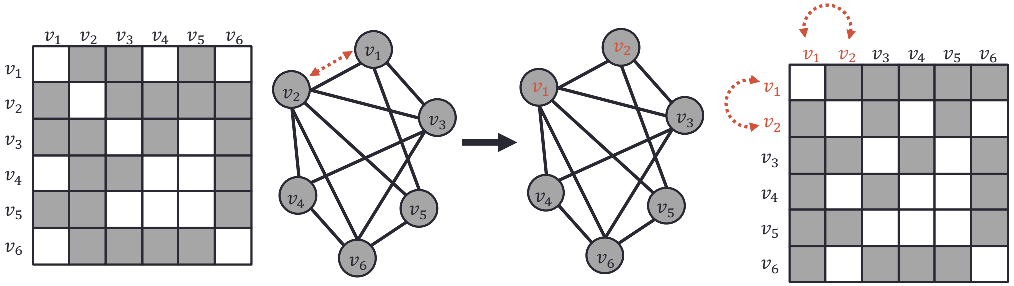
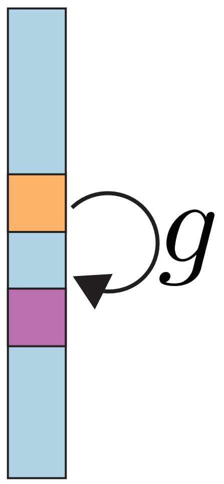


Formal definition of group action

- Let $G \leq S_n$
- $g \in G$ acts on a vector $x \in \mathbb{R}^n$ by permuting its coordinates:

$$gx = (x_{g^{-1}(1)}, \dots, x_{g^{-1}(n)})$$



Formal definition of group action

- $g \in G$ acts on a tensor $\mathrm{X} \in \mathbb{R}^{n^k}$ by permuting its coordinates in each dimension:

$$(gX)_{i_1,\dots,i_k}$$

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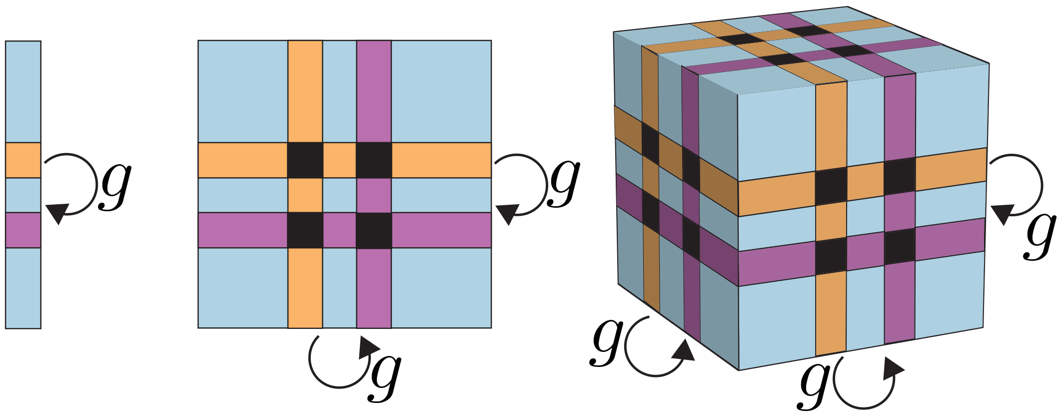
$$(gX)_{i_1,\ldots,i_k} = X_{g^{-1}(i_1),\ldots,g^{-1}(i_k)}$$



Invariant and equivariant functions

Definition: A function $f: \mathbb{R}^n \to \mathbb{R}$ is **invariant** with respect to a group G if:

$$f(gx) = f(x), \quad \forall g \in G$$

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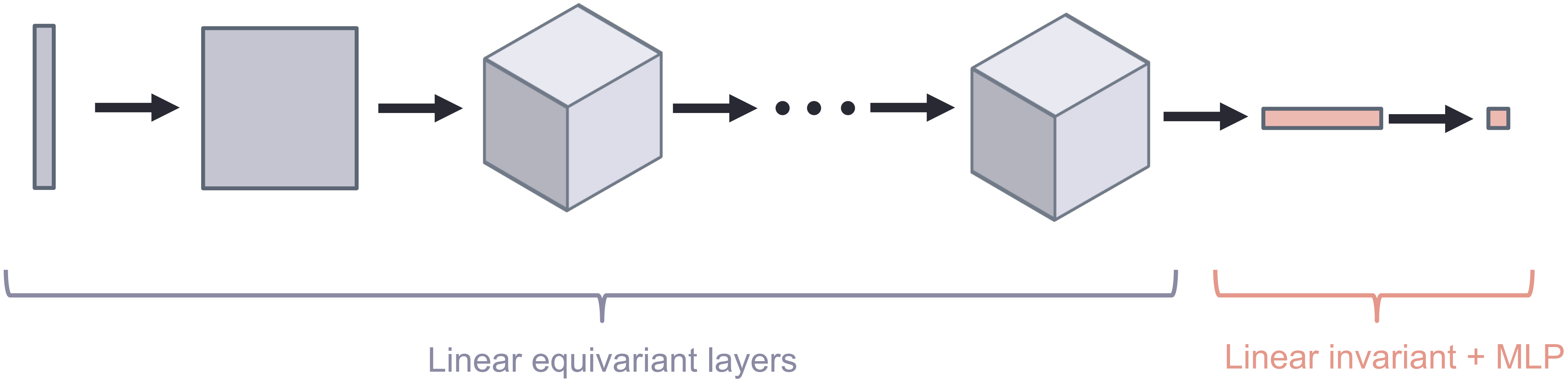
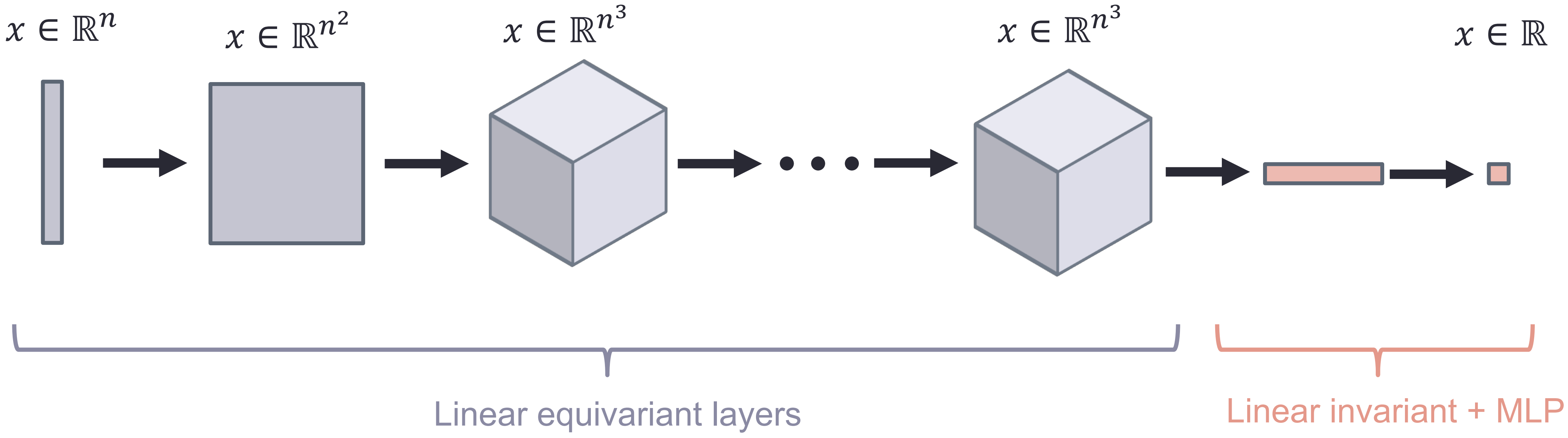
G-invariant networks



G-invariant networks



Main question:

How expressive are G-invariant networks?

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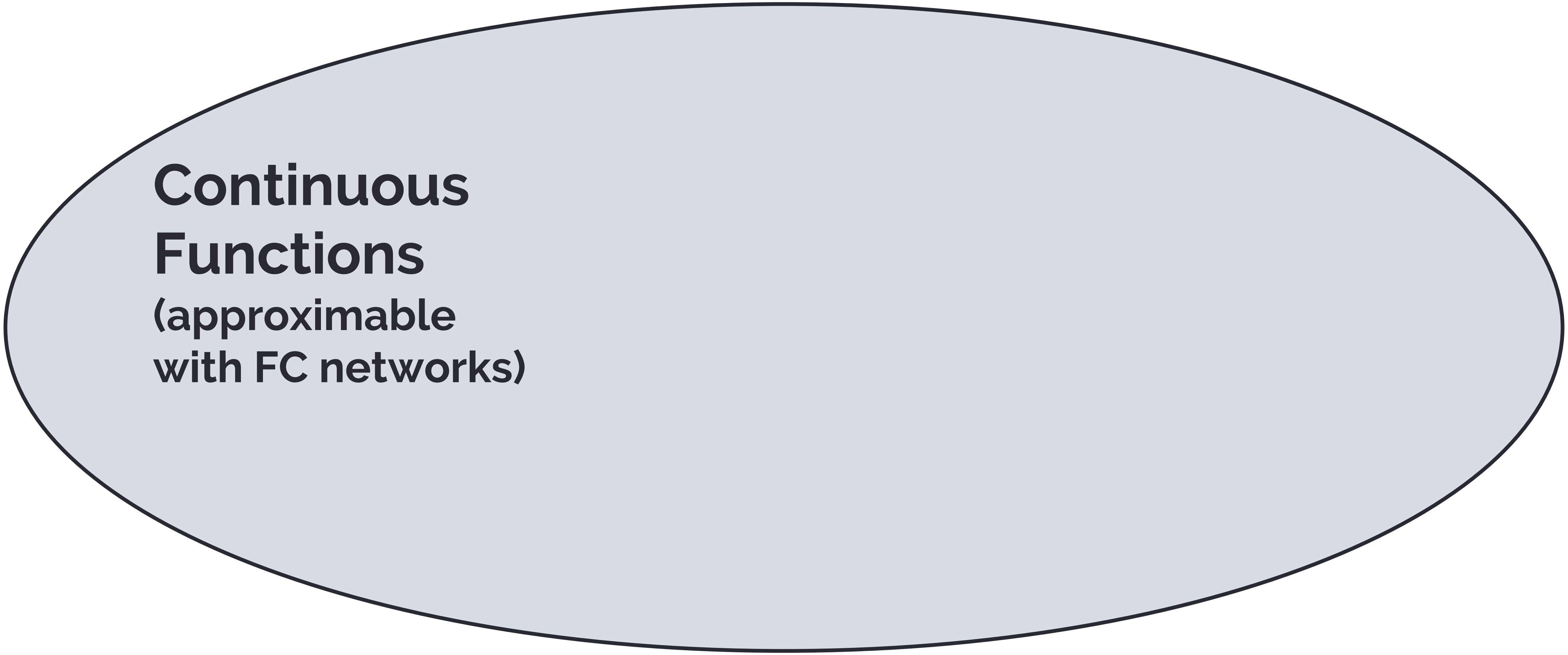

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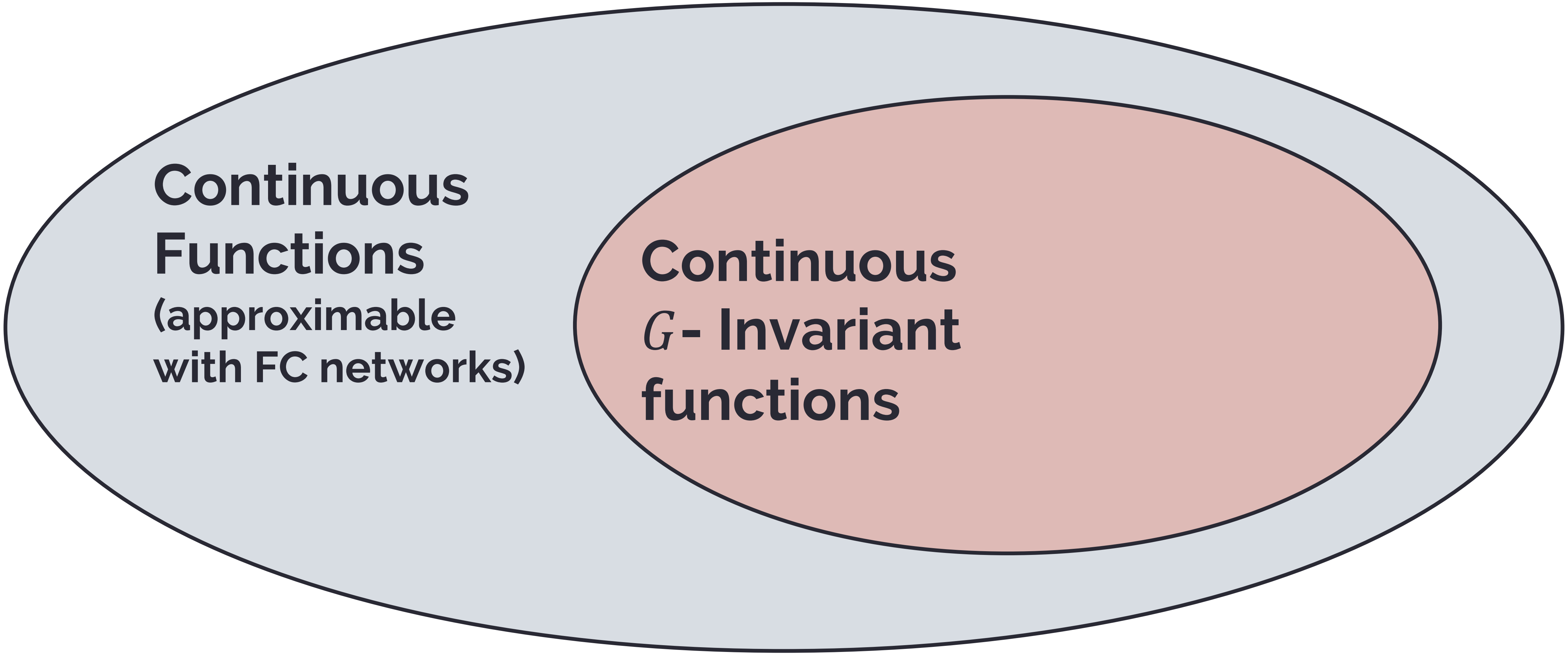

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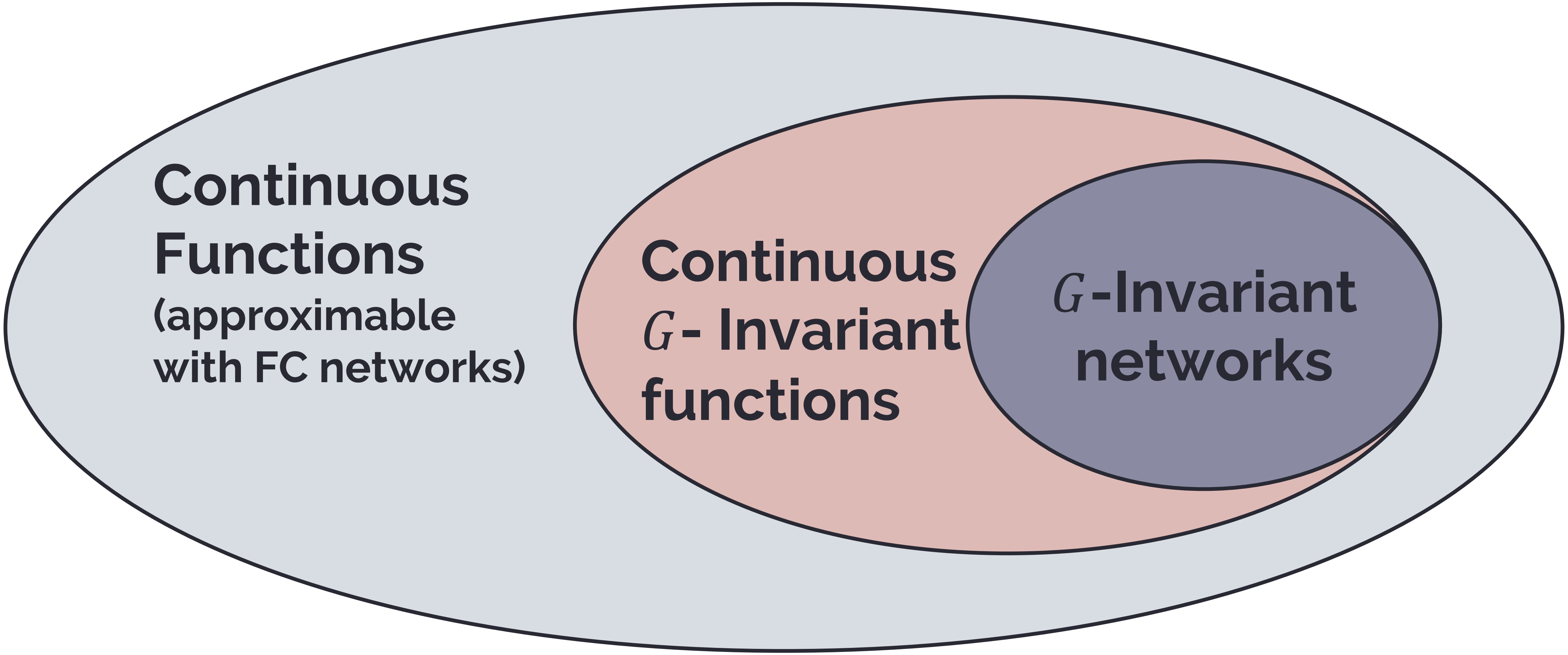

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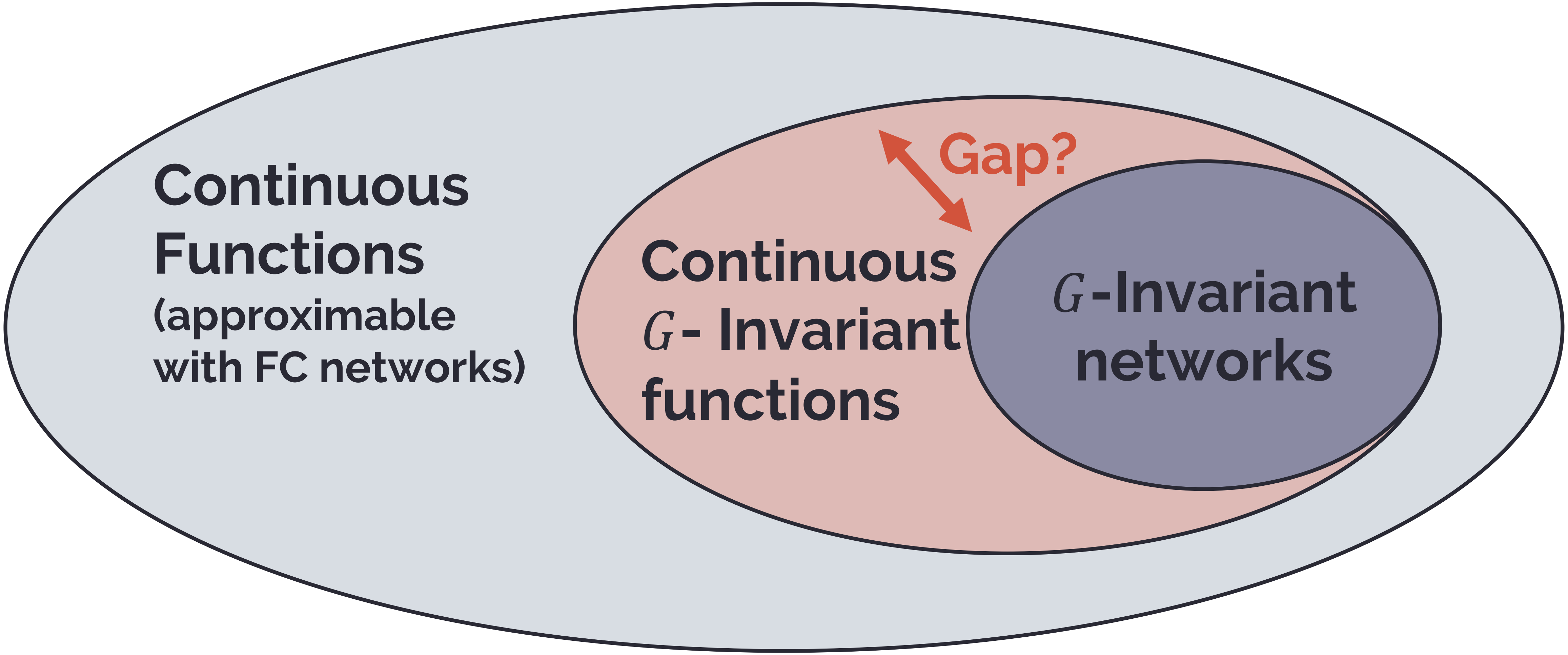
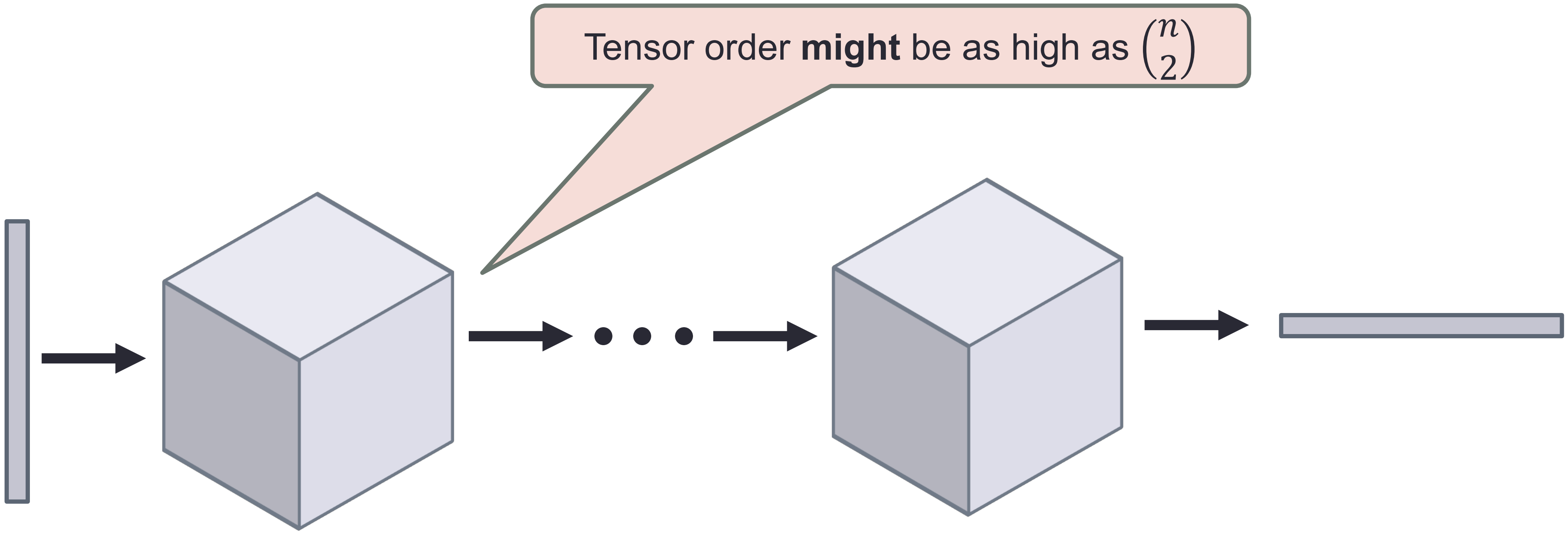


Theoretical results

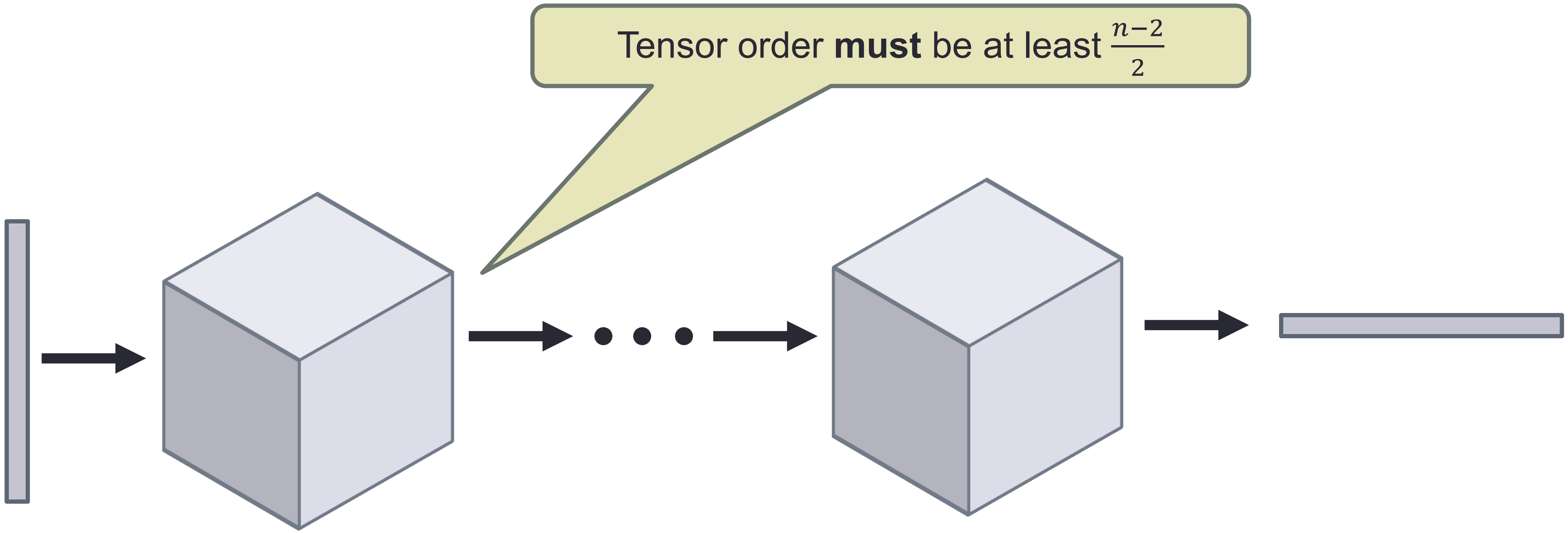
Universality of high-order networks

Theorem 1. G-invariant networks are universal.



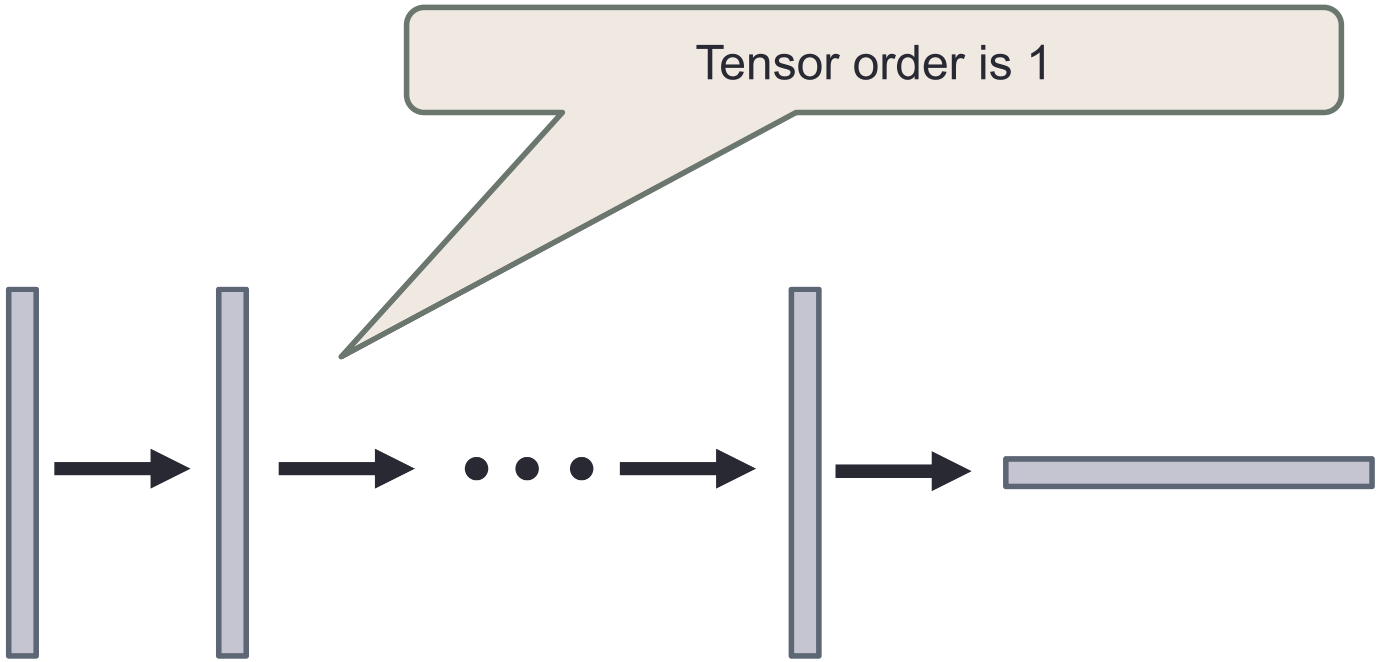
Lower bound on network order

Theorem 2. There exists groups $G \leq S_n$ for which the tensor order should be at least $O(n)$ in order to achieve universality



Necessary condition for first order networks

Theorem 3. Let $G \in S_n$. If first order G-invariant networks are universal, then $|[n]^2/H| < |[n]^2/G|$ for any strict super-group $G < H \leq S_n$.



The End

- Support
  - ERC Grant (LiftMatch)
  - Israel Science Foundation

"Invariant Graph Networks"
by Yaron Lipman
Saturday 11am, Grand Ballroom B
Learning and Reasoning with Graph-Structured Representations workshop

- Thanks for listening!