Graph Sketching, Streaming and Space Efficient Optimization
Part II

Sudipto Guha and Andrew McGregor

Space Efficient Optimization for Graphs

Sampling, Connectivity, Sparsification: How do these get used?

Techniques like Dimensionality Reduction, Embeddings, $L_p \rightarrow L_q$, etc., are improving vector based computations.

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Streaming as a vehicle to organize accesses in an algorithm.

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- Embed the graph. Solve the optimization in the embedded space.
- Small space and faster runtimes!

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We focus on cut-sparsification. Will not always work out of the box.
We will have to change relaxations and use sparsification with care.

Plan of the Hour

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Fast and approximate recap of fast and approximate convex optimization.

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Multiplicative Weights Method (MWM). LP version. Oracles.

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Global (Cut)-Sparsification. Single pass.

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Example: Correlation Clustering. Max-version.

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New relaxations + oracle. Benefits in running time + space. Both cases.

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Iterative (local) (Cut)-Sparsification. Multiples passes, Batch modes.

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Example 2. Non-bipartite Matching. $(1+\epsilon)$-apx.

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Benefits in time+space+adaptivity.

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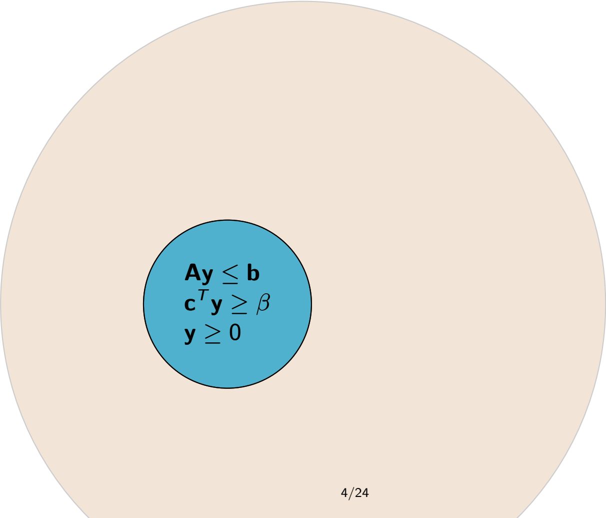
Wrap-up.

Multiplicative Weights Method: A Recap

$\mathbf{Ay} \leq \mathbf{b}$
$\mathbf{c}^T\mathbf{y} \geq \beta$
$\mathbf{y} \geq 0$

Multiplicative Weights Method: A Recap



$\mathbf{A}\mathbf{y} \leq \rho\mathbf{b}$
$\mathbf{c}^T\mathbf{y} \geq \beta$
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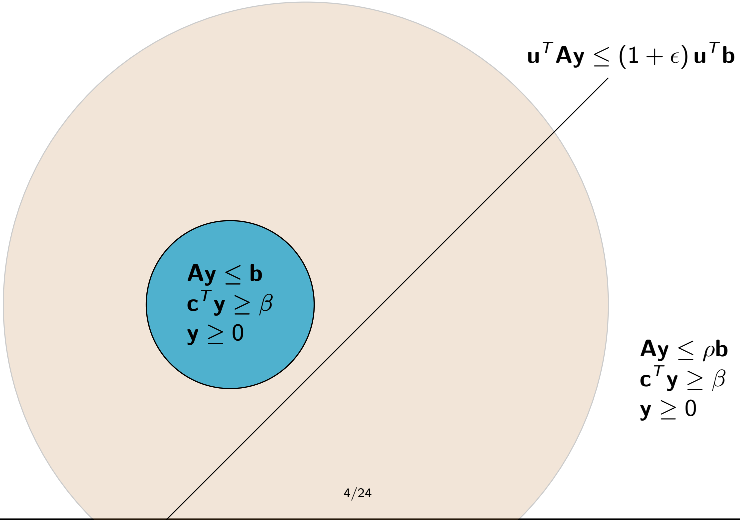
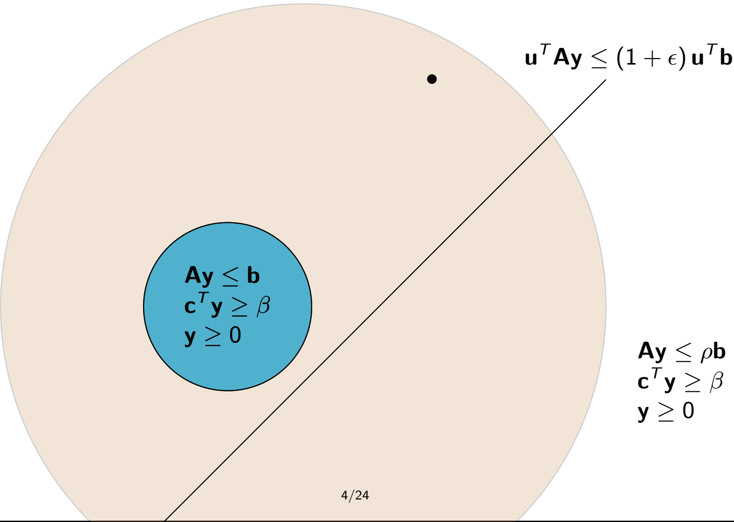
Initially $\mathbf{u} = \mathbf{1}$.



Multiplicative Weights Method: A Recap

Initially $\mathbf{u} = \mathbf{1}$.
If $\mathbf{A}_i\mathbf{y} < \mathbf{b}_i$: lower $\mathbf{u}_i$, i.e., $\mathbf{u}_i \leftarrow \mathbf{u}_i(1-\epsilon)^{(\mathbf{b}_i - \mathbf{A}_i\mathbf{y})/\mathbf{b}_i\rho}$. (Assume $\mathbf{A}, \mathbf{b} \geq \mathbf{0}$).
If $\mathbf{A}_i\mathbf{y} > \mathbf{b}_i$: raise $\mathbf{u}_i$, i.e., $\mathbf{u}_i \leftarrow \mathbf{u}_i(1+\epsilon)^{(\mathbf{A}_i\mathbf{y} - \mathbf{b}_i)/\mathbf{b}_i\rho}$.



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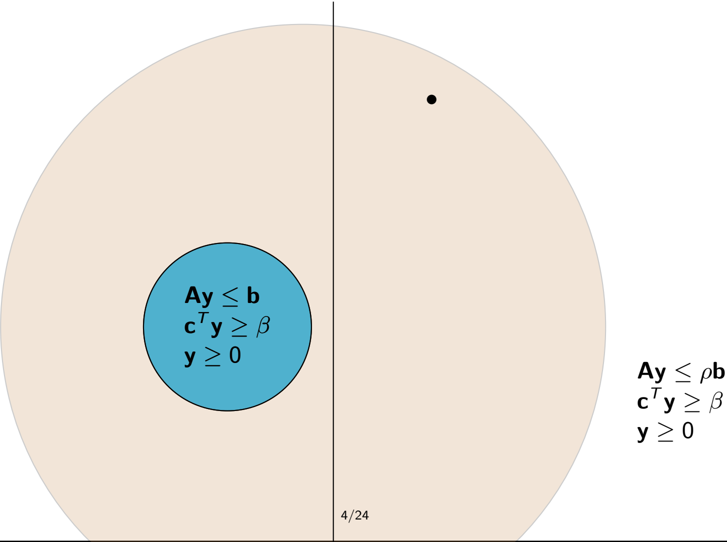

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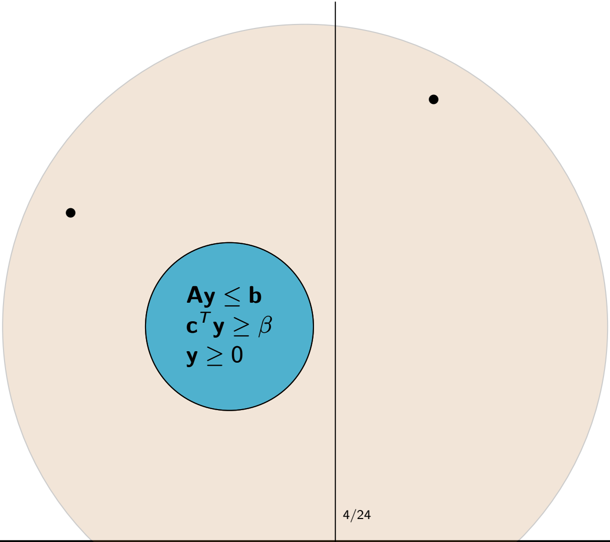

$\mathbf{A}\mathbf{y} \leq \rho\mathbf{b}$
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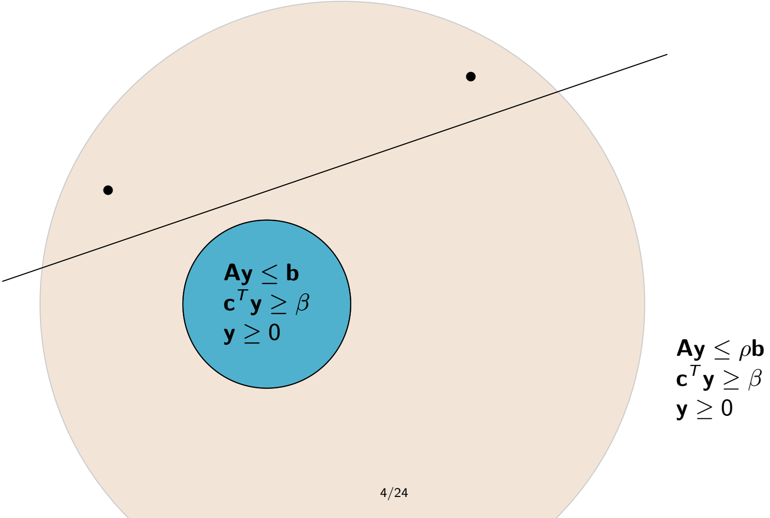

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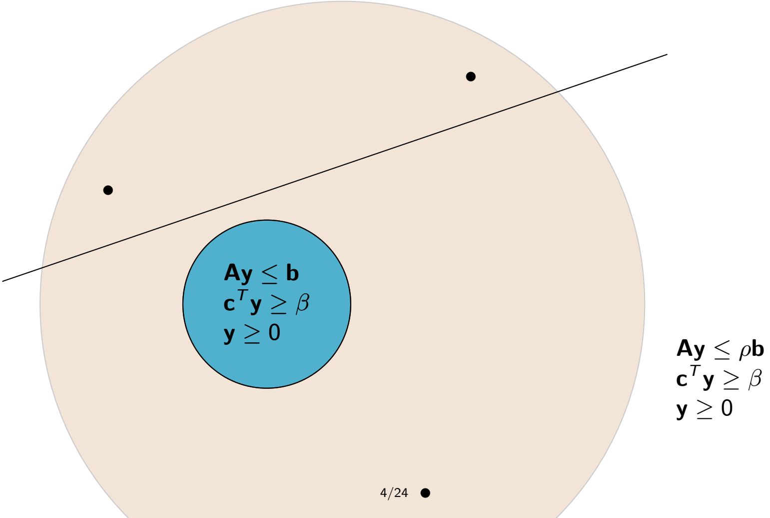

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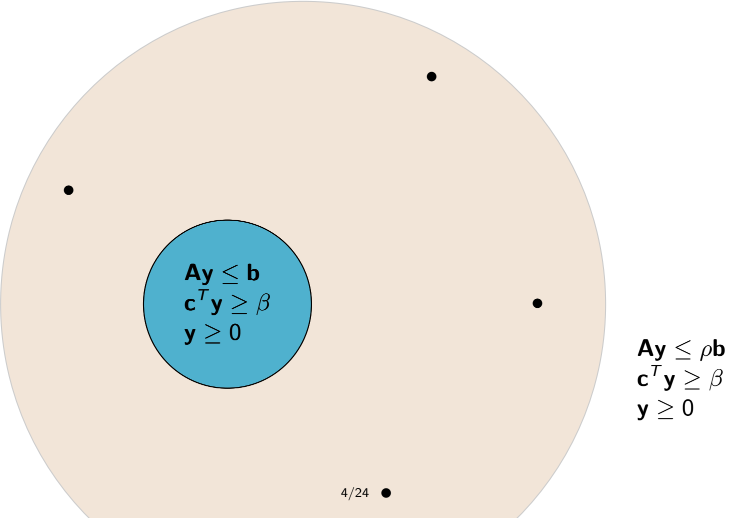

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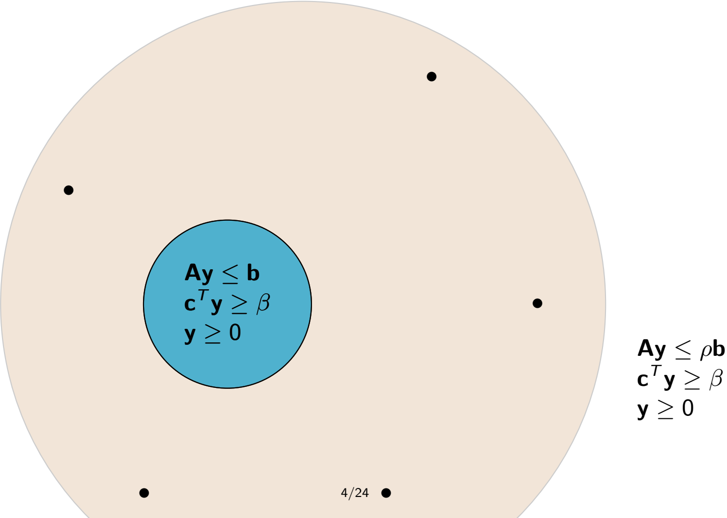

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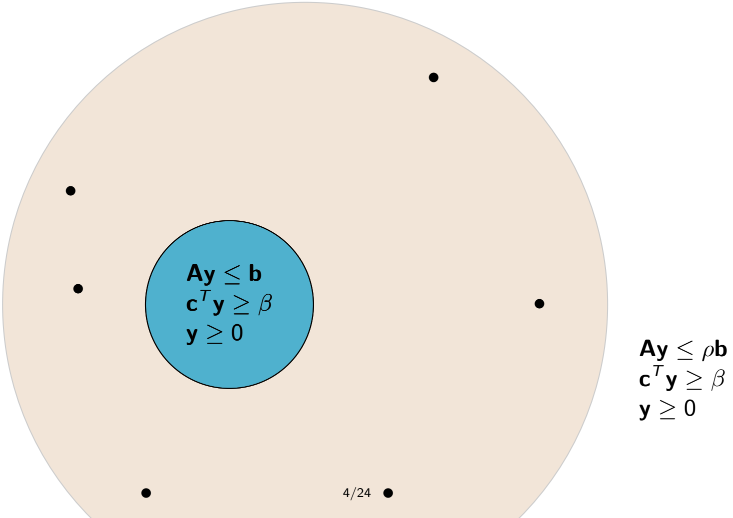

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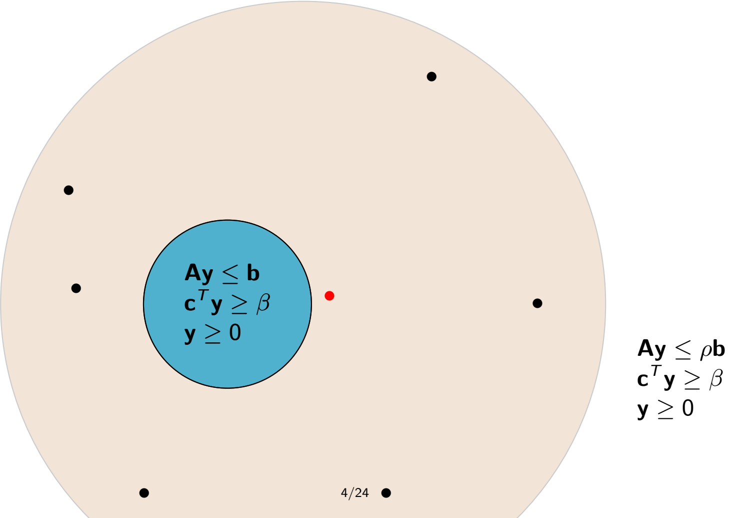

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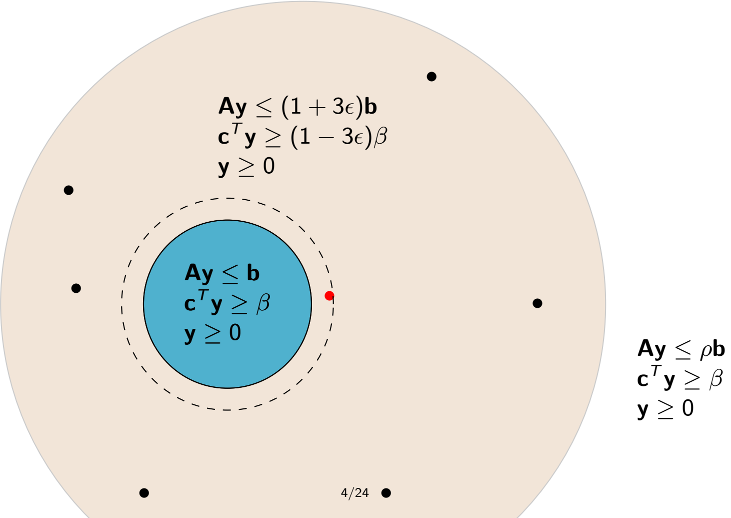
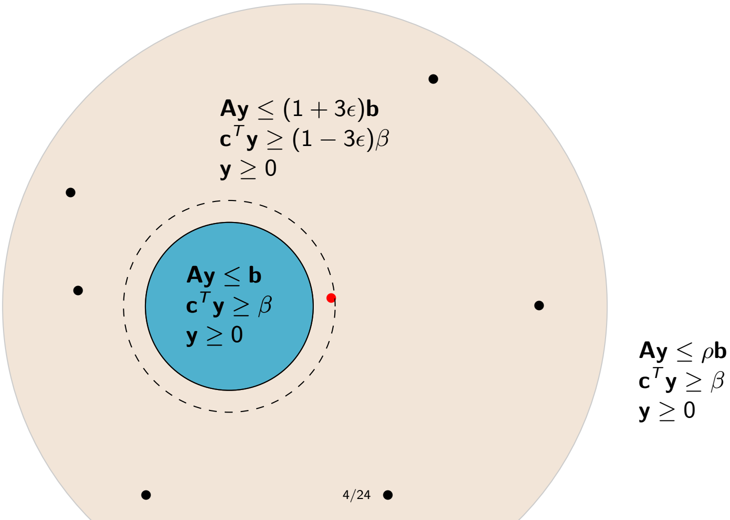


Multiplicative Weights Method: A Recap

Number of rounds depends on ρ, ϵ and other specifics of updating **u**.
$\rho =$**width**.



MWM on Streams: Bipartite Matching

Ahn, Guha 14.

Integer and fractional optimums coincide. ($y_{ij} = y_{ji}$, (i,j) implies $\in E$.)

$$\begin{aligned} \max \quad & \sum_{(i,j)} y_{ij} w_{ij} & & \\ & \sum_{j} y_{ij} & \leq 1 & \quad \forall i \\ & y_{ij} & \geq 0 & \quad \forall (i,j) \end{aligned}$$

Streams: arbitrary list of m edges, $\ldots, \langle i, j, w_{ij} \rangle, \ldots$ for an n node graph.
Different from online learning. Input itself is in small pieces.

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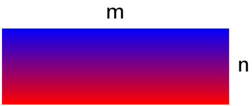

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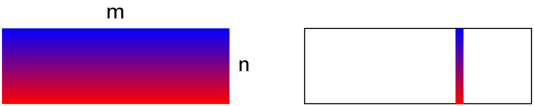

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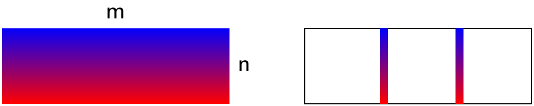

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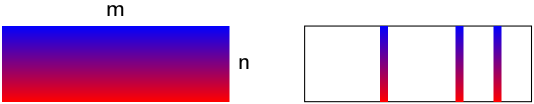

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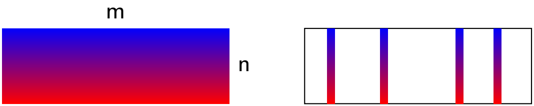

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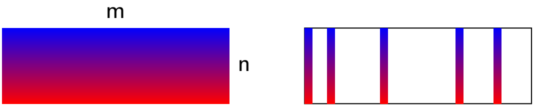

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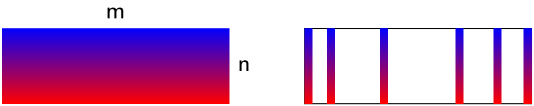

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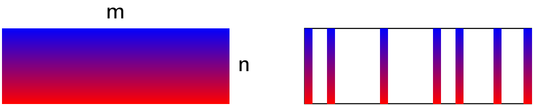

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$$
\begin{aligned}
\sum_{(i,j)} y_{ij} w_{ij} &\geq \beta \\
\sum_{j} y_{ij} &\leq 1 \quad \forall i \\
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\end{aligned}
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Streams: arbitrary list of m edges, $\ldots, \langle i, j, w_{ij} \rangle, \ldots$ for an n node graph.

Applying MWM: Point = candidate set of edges, in m-dim space.
Hyperplanes?

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Hyperplanes? $\sum_i u_i \sum_j y_{ij} \leq \sum_i u_i \quad \Leftrightarrow \quad \sum_{(i,j)} y_{ij}(u_i + u_j) \leq \sum_i u_i$.
Store & update $\mathbf{u}$. $O(n)$ storage.

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$$
\text{Want: } \left\{
\begin{array}{lll}
\sum_{(i,j)} y_{ij}(u_i + u_j) \sum_i u_i & \leq \sum_i u_i & \\
\sum_{(i,j)} y_{ij} w_{ij} & \geq \beta & \\
\sum_j y_{ij} & \leq \rho & \forall i \\
y_{ij} & \geq 0 & \forall (i,j)
\end{array}
\right. .
$$

MWM on Streams: Bipartite Matching

$$\text{Want: } \begin{cases} \sum_{(i,j)} y_{ij}(u_i + u_j) & \leq \sum_i u_i \\ \sum_{(i,j)} y_{ij} w_{ij} & \geq \beta \\ \sum_j y_{ij} & \leq \rho \quad \forall i \\ y_{ij} & \geq 0 \quad \forall (i,j) \end{cases}$$

,

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$$\text{Now } \exists \mathbf{y}, \;\; \forall \lambda \geq 0 \quad \begin{cases} \sum_{(i,j)} (w_{ij} - \lambda(u_i + u_j)) y_{ij} & \geq (\beta - \lambda \sum_i u_i) \\ \sum_{j} y_{ij} & \leq 1 \qquad \forall i \\ y_{ij} & \geq 0 \quad \forall (i,j) \end{cases}$$

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Oracle(λ):

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Now $\exists \mathbf{y}, \ \forall \lambda \geq 0$ $\begin{cases} \sum_{(i,j)} (w_{ij} - \lambda(u_i + u_j)) y_{ij} & \geq (\beta - \lambda \sum_i u_i) \\ \sum_j y_{ij} & \leq 1 \quad \forall i \\ y_{ij} & \geq 0 \quad \forall (i,j) \end{cases}$

Oracle(λ):

- Seeing (i,j) compute $(w_{ij} - \lambda(u_i + u_j))$. If -ve, discard.

MWM on Streams: Bipartite Matching

Want: $$\begin{cases} \sum_{(i,j)} y_{ij}(u_i + u_j) & \leq \sum_i u_i \\ \sum_{(i,j)} y_{ij} w_{ij} & \geq \beta \\ \sum_j y_{ij} & \leq \rho \quad \forall i \\ y_{ij} & \geq 0 \quad \forall (i,j) \end{cases}$$

Have y, $\forall \lambda \geq 0$ $$\begin{cases} \sum_{(i,j)} (w_{ij} - \lambda(u_i + u_j))\, y_{ij} & \geq (\beta - \lambda \sum_i u_i)/c \\ \sum_j y_{ij} & \leq 1 \quad \forall i \\ y_{ij} & \geq 0 \quad \forall (i,j) \end{cases}$$

Oracle(λ):

- Seeing (i,j) compute $(w_{ij} - \lambda(u_i + u_j))$. If -ve, discard.
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If Oracle(λ) for $\lambda = 0$ satisfies $\sum_{(i,j)} y_{ij}(u_i + u_j) \leq \sum_i u_i / c$ then we also have: $\sum_{(i,j)} w_{ij} y_{ij} \geq \beta / c$. **(easier case)**

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Multiply **y** by c. Set $\rho = c$ and we have a solution!

MWM based Bipartite Matching for Map-Reduce?

More general than streaming.

Map-Reduce based 8 approximations in $O(\log n)$ rounds exist, e.g., Lattanzi, Mosely, Suri, Vassilivitskii 11.

We can compose them. $O(\log n)$ rounds to get a c-approximation. Repeat $O(c\epsilon^{-2} \log n)$ times to get a $(1+\epsilon)$- fractional solution.

Can also round to an integral solution in small space.
A story for some other time.

Up Next ...

Fast and approximate recap of fast and approximate convex optimization.
Multiplicative Weights Method (MWM). LP version. Oracles.
Example: Bipartite Matching. $(1+\epsilon)$-apx. MWM on Streams.

Global (Cut)-Sparsification. Single pass.

(a) Multiplicative Weights Method on SDPs.
Example: Correlation Clustering. Max-version.

(b) Multiplicative Weights Method on LPs.
Example: Correlation Clustering. Min-version.

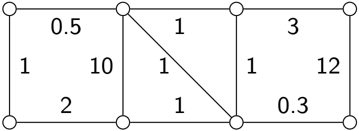
New relaxations + oracle. Benefits in running time + space. Both cases.

Iterative (local) (Cut)-Sparsification. Multiples passes, Batch modes.
Example 2. Non-bipartite Matching. $(1+\epsilon)$-apx.
Cornerstone of Combinatorial Optimization, Dantzig Decompositions.
Benefits in time+space+adaptivity.

Wrap-up.

Global Sparsification: There and back again

Think of a problem on graph cuts.



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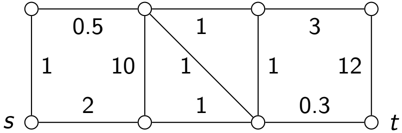

Min s-t Cut?

Sparsification preserves all cuts within $(1 \pm \epsilon)$.

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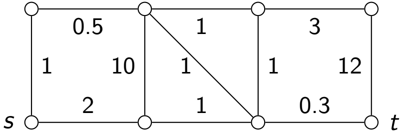

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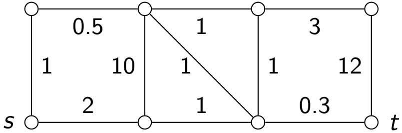

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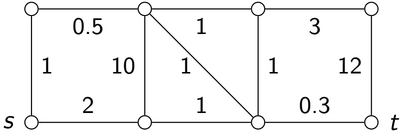

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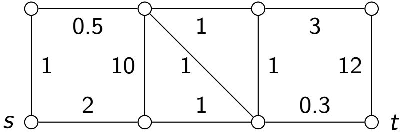

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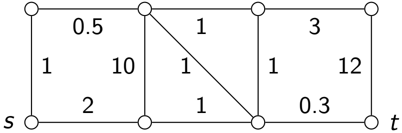

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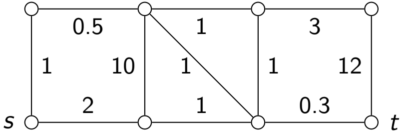

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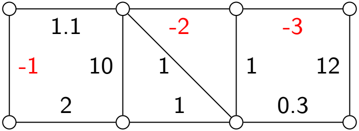
We will see examples both (a)–(b) and how to overcome them.
Lets consider a variant of **clustering.** And richer graphs.

Correlation Clustering



Find a grouping that **agrees** most with the graph.

- Count +ve edges in clusters. Count -ve edges **out** of clusters.
- Use as many clusters as you like.

Alternatively we can find a grouping that **disagrees** least.

NP Hard. Bansal Blum, Chawla, 04.

Many approximation algorithms are known. For many variants.
The approximations we see here were known defore, we will not focus on the factor.

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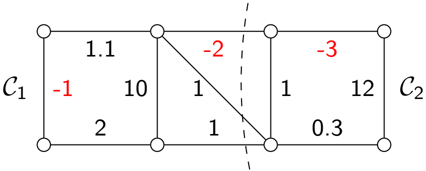

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Tutorial in KDD 2014. Bonchi, Garcia-Soriano, Liberty.
Clustering of objects known only through relationships.
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Likely **Mr Smith** $\neq$ **Mrs Smith**. Large -ve weight.
The other references can be either. Small weights depending on context.
Weights are not a metric. Have a large range.

Max-Agreement and SDPs

$x_{ij} = 1$ if in same group, and 0 otherwise. $E(+/-) = +/-$ve edge sets.
Think of vector programming over unit length vectors. $x_{ij} = v_i \cdot v_j \leq 1$.

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Why??

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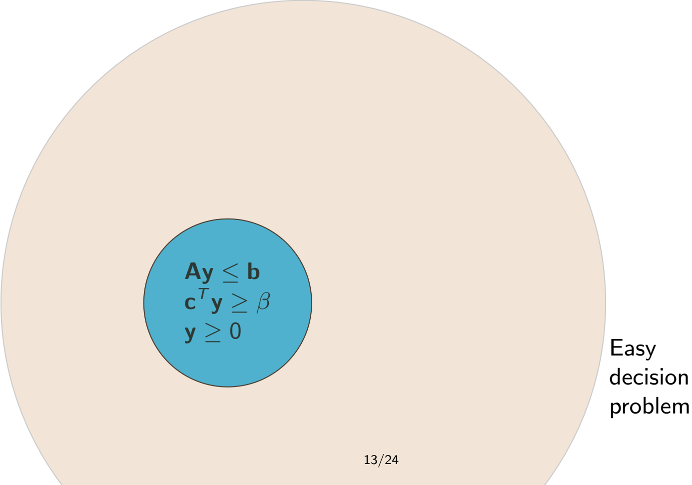

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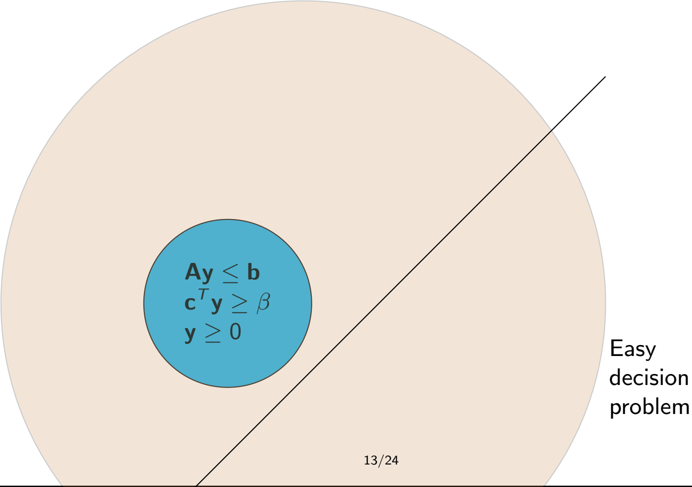

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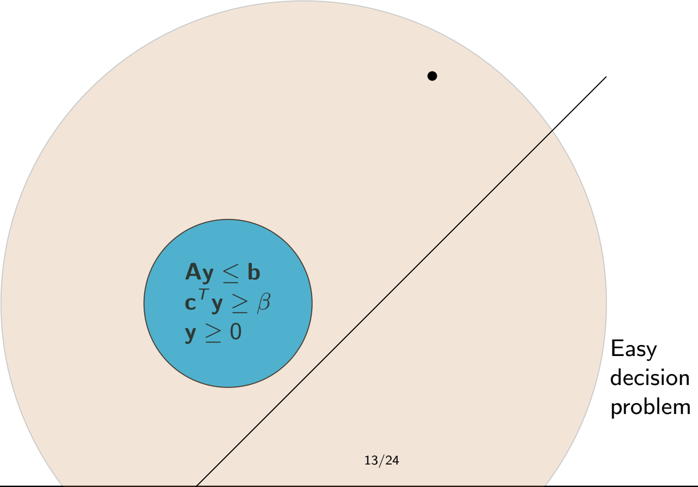

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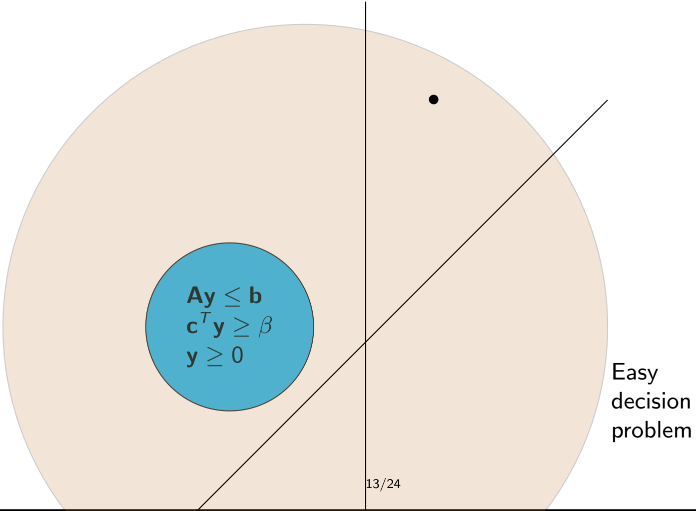

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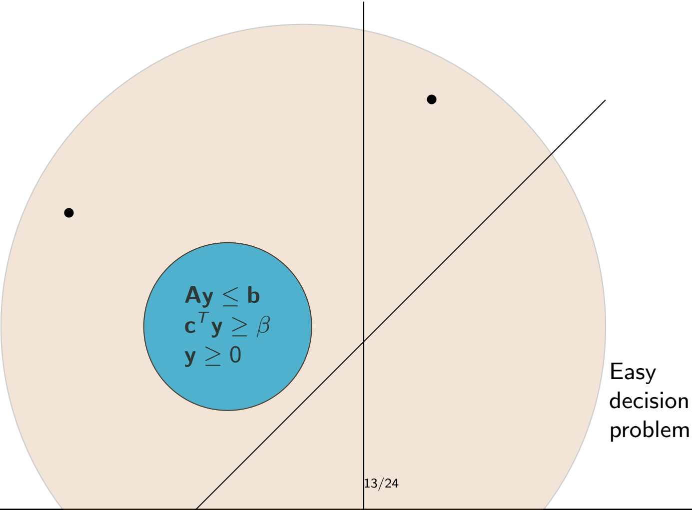

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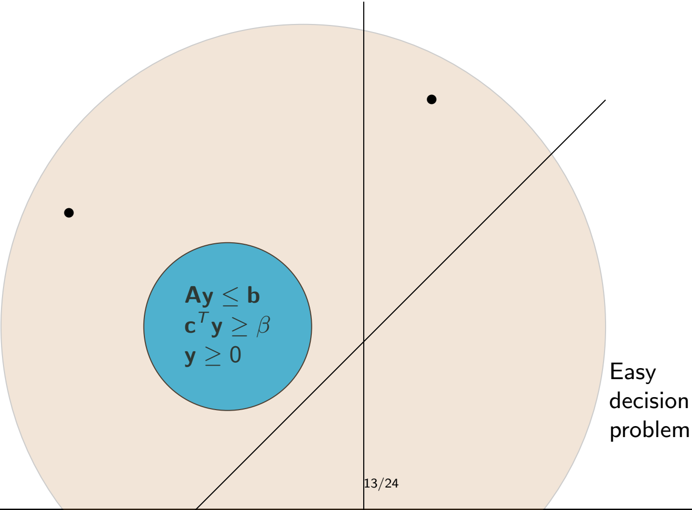

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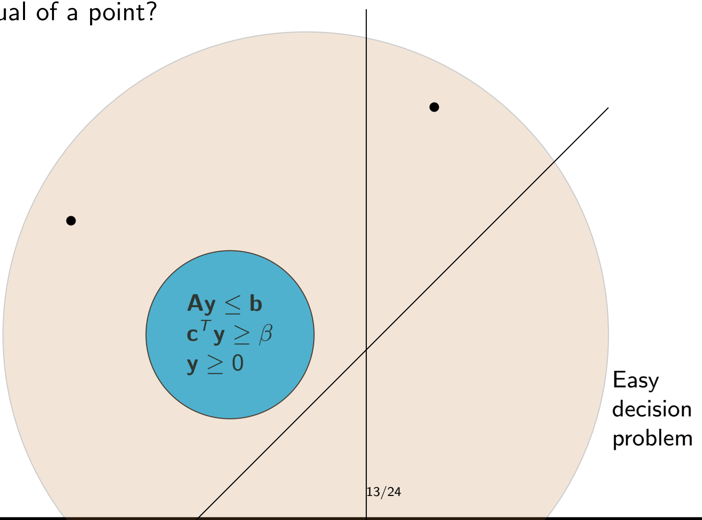

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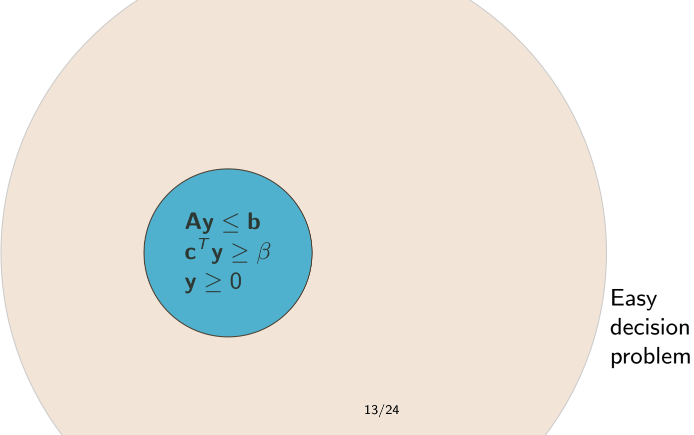

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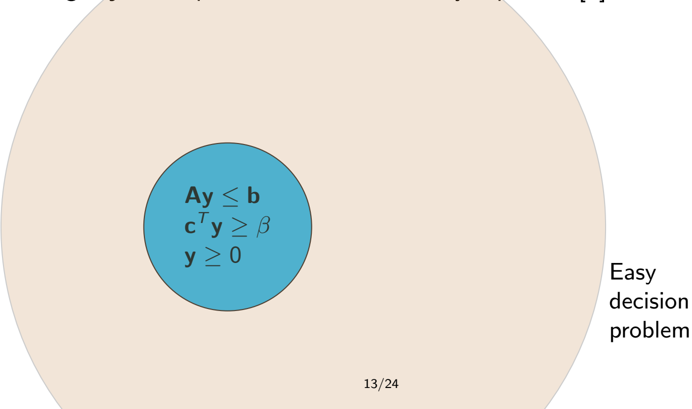

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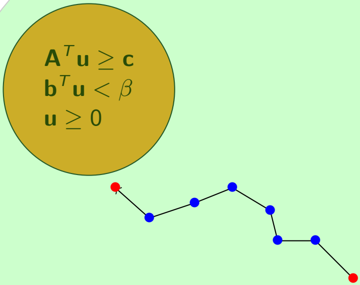
Think trajectories.
MWM on dual.
e.g., Steurer 10.



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Why. ~~Does not work (width is high).~~ Linear Space. Linear time. 0.76-apx
Relaxation needs to be compatible with trajectory.
Single pass. Sparsify $E(+)$ and $E(-)$ separately.

Ahn 13, Ahn, Cormode, Guha, McGregor, Wirth 15.

Min-Disagreement

Equivalent to Max-Agreement at optimality. Not in approximation.

$x_{ij} = 1$ if in same group, and 0 otherwise. $E(+/-) = +/-$ve edge sets.

$$
\begin{aligned}
&\min \sum_{(i,j)\in E(+)} w_{ij}(1 - x_{ij}) + \sum_{(i,j)\in E(-)} |w_{ij}| x_{ij} && \\
&x_{ij} \le 1 && \forall i, j \\
&x_{ij} \ge 0 && \forall i, j \\
&(1 - x_{ij}) + (1 - x_{jk}) \ge (1 - x_{ik}) && \forall i, j, k
\end{aligned}
$$

A linear program.

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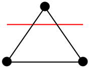

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\end{aligned}$$

Triangle constraints

A linear program. $\Theta(n^3)$ Constraints, $\Theta(n^2)$ variables.

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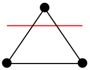

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Triangle constraints

A linear program. $\Theta(n^3)$ Constraints, $\Theta(n^2)$ variables.
1 pass lower bound of $|E(-)|$ for any apx via Communication Complexity.

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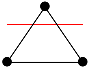

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Triangle constraints

A linear program. $\Theta(n^3)$ Constraints, $\Theta(n^2)$ variables.
1 pass lower bound of $|E(-)|$ for any apx via Communication Complexity.

Sparsify $E(+)$, store $E(-)$? Will have $\tilde{O}(n) + |E(-)|$ variables.

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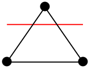

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A linear program. $\Theta(n^3)$ Constraints, $\Theta(n^2)$ variables.
1 pass lower bound of $|E(-)|$ for any apx via Communication Complexity.

Sparsify $E(+)$, store $E(-)$? Will have $\tilde{O}(n) + |E(-)|$ variables.

Does **not** work. The triangle constraints need all $\binom{n}{2}$ variables.

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$$\begin{aligned} \min \quad & \sum_{(i,j)\in E(+)} w_{ij} y_{ij} + \sum_{(i,j)\in E(-)} |w_{ij}| z_{ij} & \\ & y_{ij}, z_{ij} \geq 0 & \forall i,j \\ & y_{ij}, z_{ij} ? & \end{aligned}$$

Sparsify $E(+)$. Store $E(-)$. $\Theta(n^2) \rightarrow \tilde{O}(n) + |E(-)|$ variables?
$\Theta(n^3)$ Constraints

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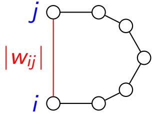

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$$y_{ij}, z_{ij} \geq 0 \qquad \forall i, j$$

$$\sum_{(u,v)\in P(ij)} y_{uv} + z_{ij} \geq 1 \qquad \forall i, j, \text{and } i\text{-}j \text{ path } P(ij)$$

Sparsify $E(+)$. Store $E(-)$. $\Theta(n^2) \to \tilde{O}(n) + |E(-)|$ variables.
$\Theta(n^3)$ Constraints $\to$ Exponentially many constraints!

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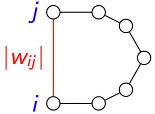

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Solve LP (ellipsoid) & **Ball Growing**: Garg, Vazirani, Yannakakis 93.

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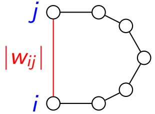

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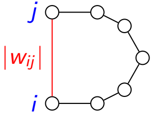

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MWM on the dual. $\tilde{O}(n + |E(-)|)$ space and $\tilde{O}(n^2)$ time. ACGMW 15.
Round infeasible primal (the running average). Success $\rightarrow$ done.
Failure $\rightarrow$ violated constraint(s) $\rightarrow$ point needed for MWM on Dual.

Up Next ...

Fast and approximate recap of fast and approximate convex optimization.
Multiplicative Weights Method (MWM). LP version. Oracles.
Example: Bipartite Matching. MWM on Streams.

Global (Cut)-Sparsification. Single pass.

(a) Multiplicative Weights Method on SDPs.
Example: Correlation Clustering. Max-version.

(b) Multiplicative Weights Method on LPs.
Example: Correlation Clustering. Min-version.

New relaxations + oracle. Benefits in running time + space. Both cases.

Iterative (local) (Cut)-Sparsification. Multiples passes, Batch modes.
Example 2. Non-bipartite Matching. $(1+\epsilon)$-apx.
Cornerstone of Combinatorial Optimization, Dantzig Decompositions.
Benefits in time+space+adaptivity.

Wrap-up.

New Strategy: Putting the Horse before the Cart

A **natural** algorithm for non-bipartite matching. Ahn, Guha 15.

1. Find an initial solution.
2. We assign some prices to the edges.
3. For $O(10/\epsilon)$ steps:
   3.1 Sample $n^{1.1}$ edges using current prices.
   3.2 Find the best weighted matching in the sample.
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Update:

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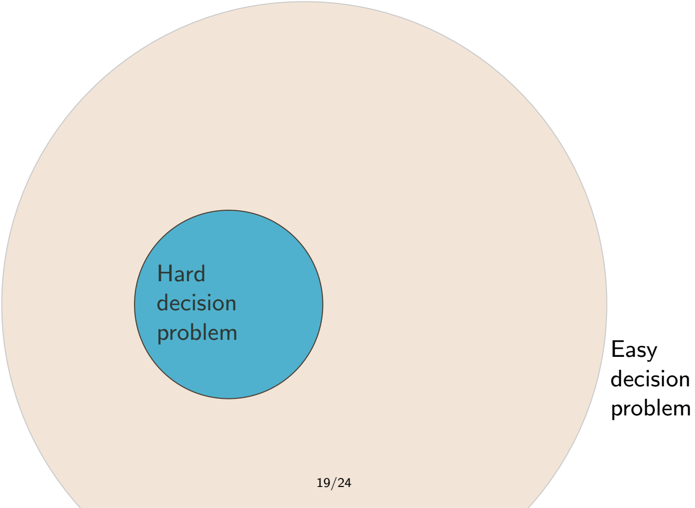
$\mathbf{u}_{ij}$: the weight in MWM corresponding to constraint (i, j) of the dual. signals if the edge relevant/not. **Sparsify those.**

Sparsification reveals a **subgraph** containing a near optimal solution. But only at near-optimality.

Dantzig Decompositions

A running average view (primal space).



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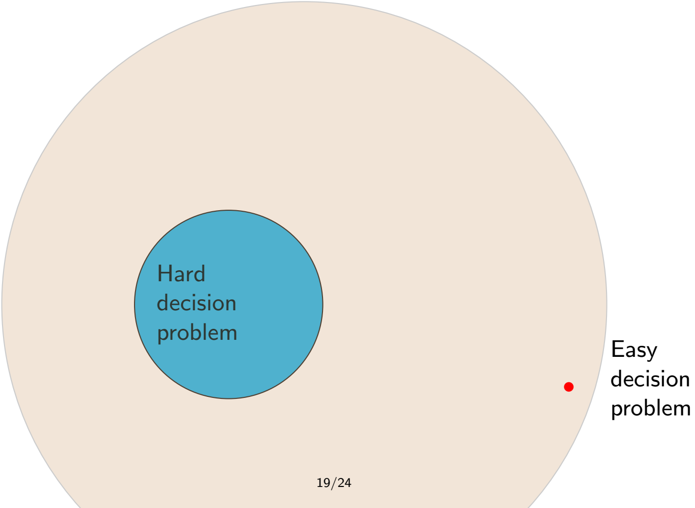

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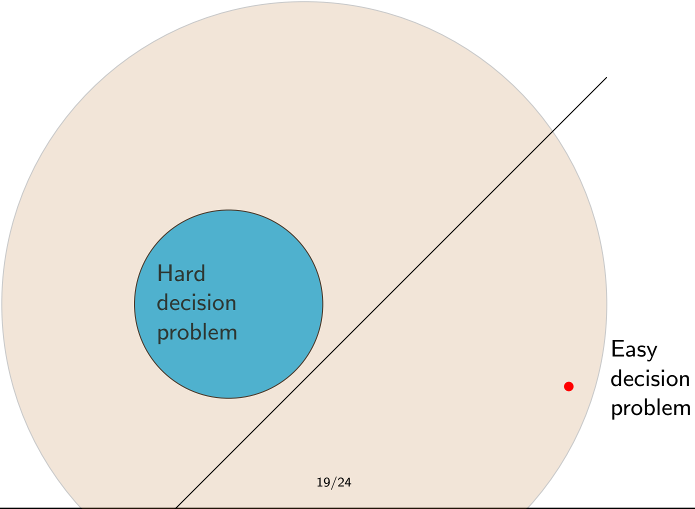

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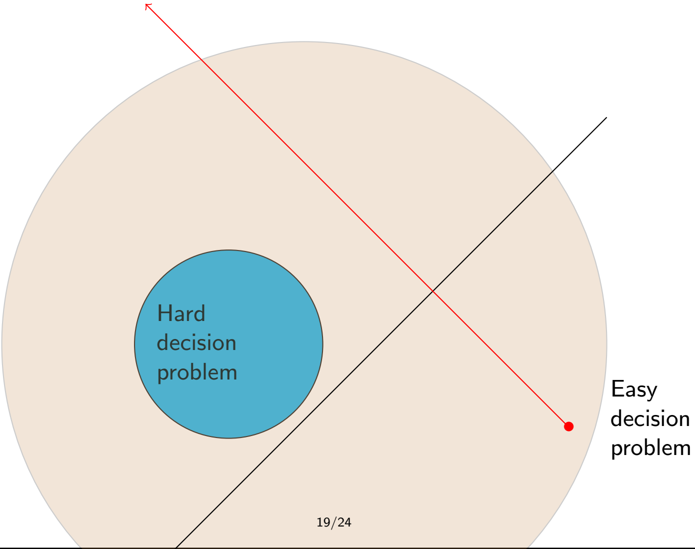

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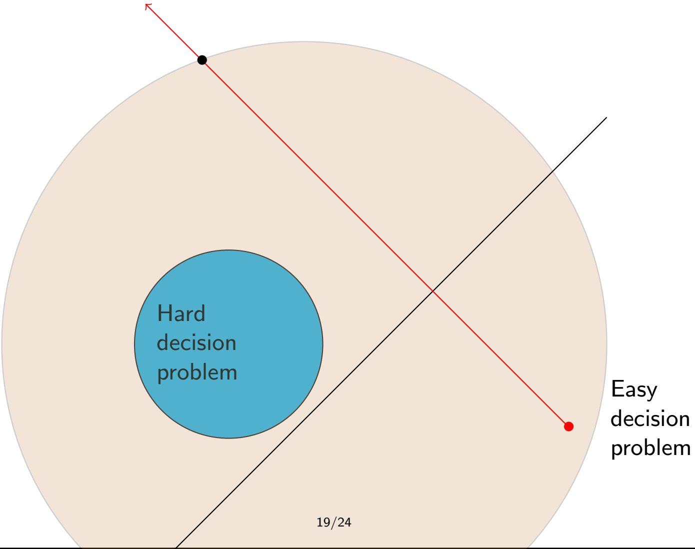

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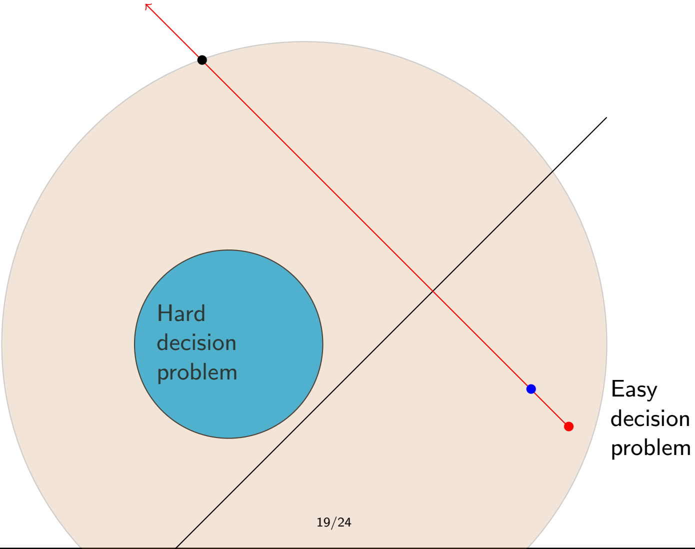

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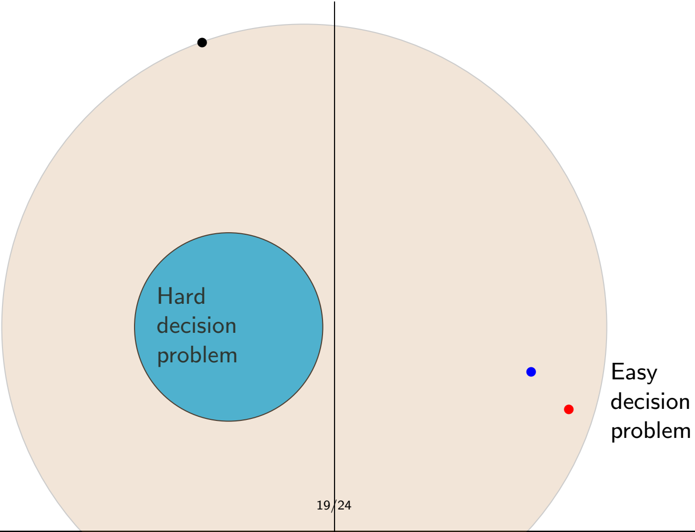

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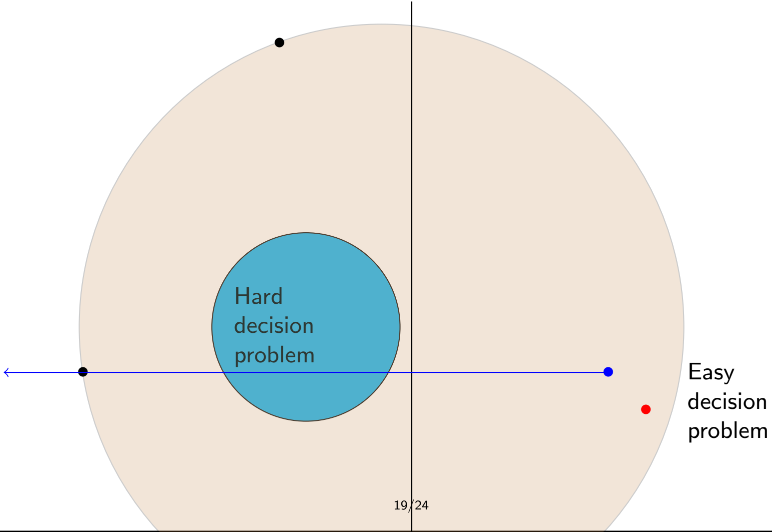

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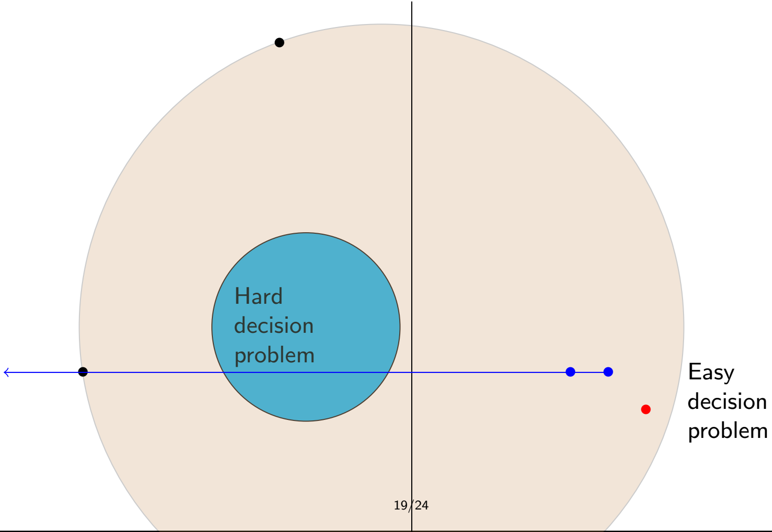

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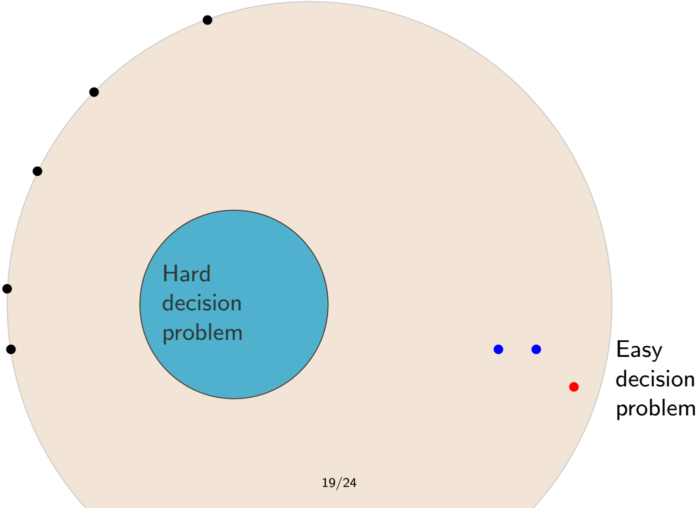

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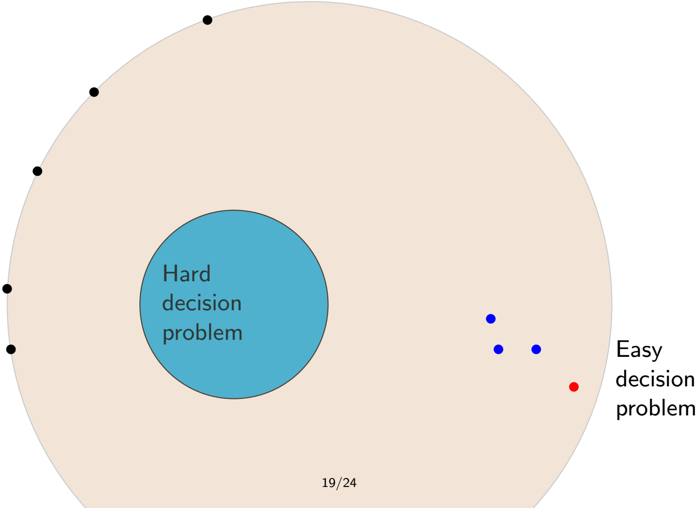

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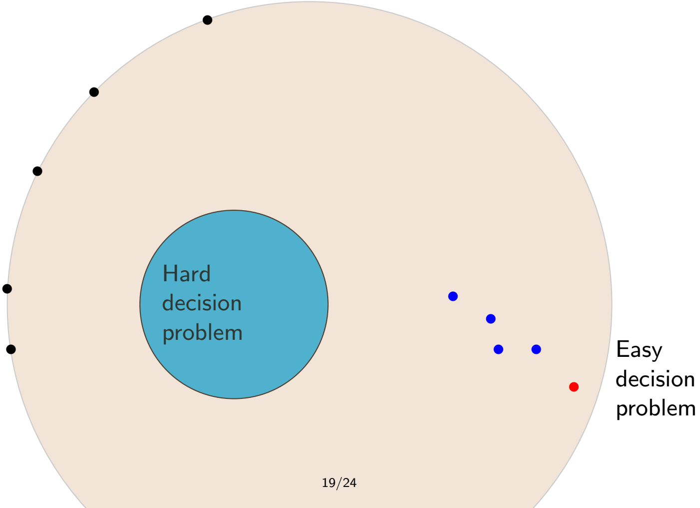

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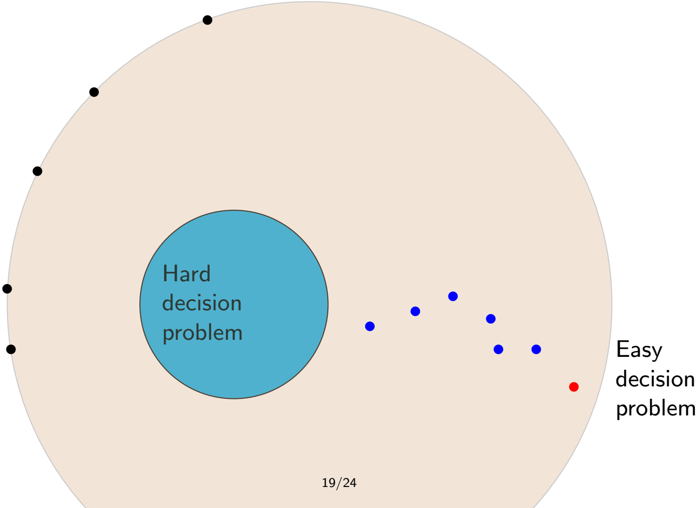

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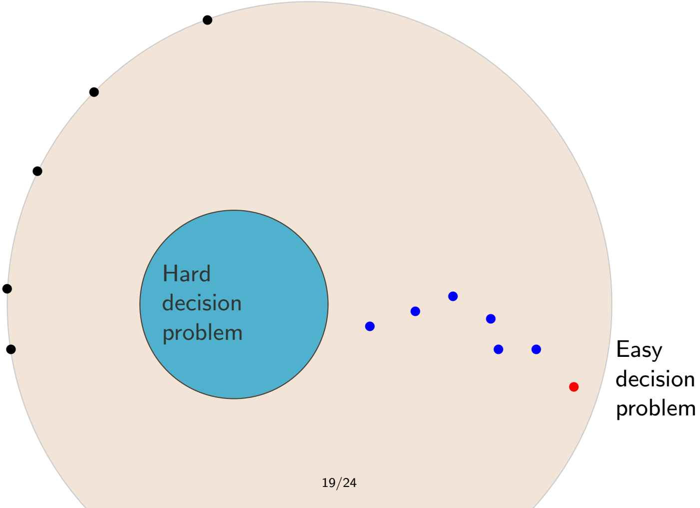

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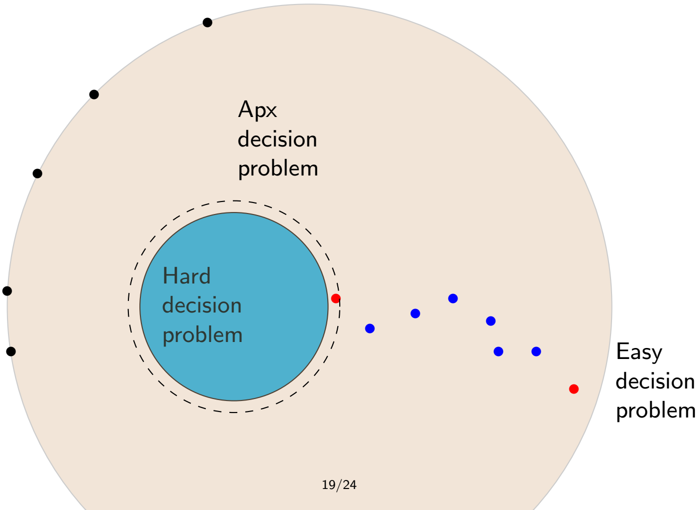

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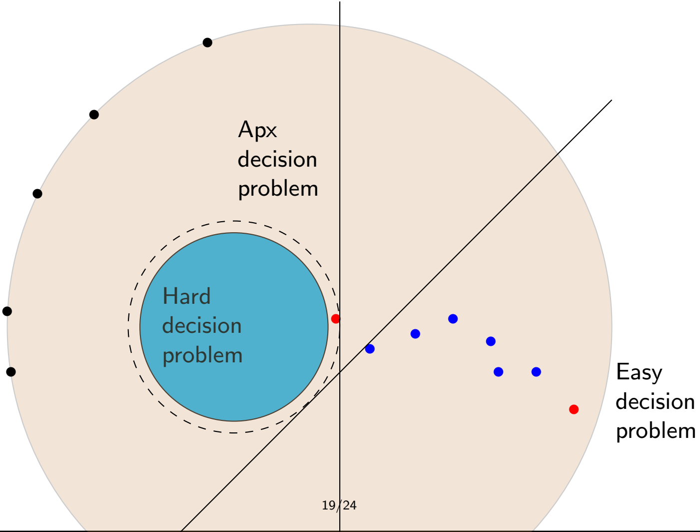

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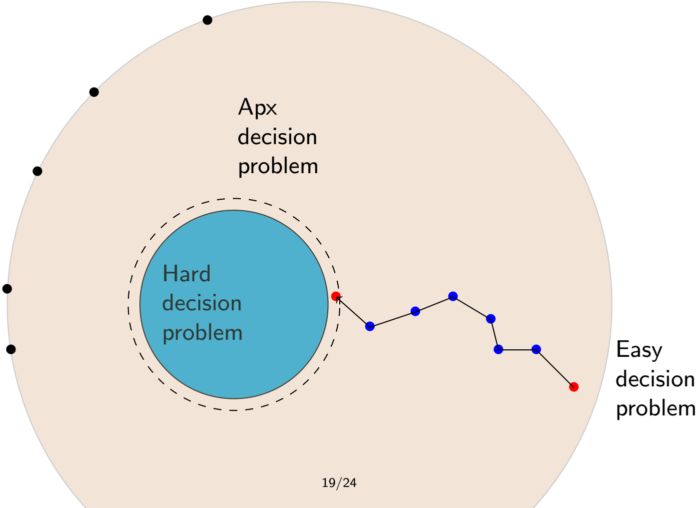
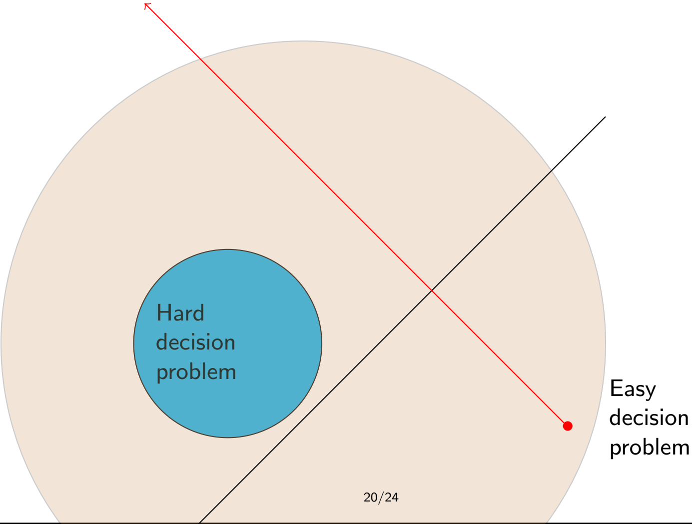


Sparsifications and Dantzig Decompositions

What if we sparsify $\mathbf{u}$?



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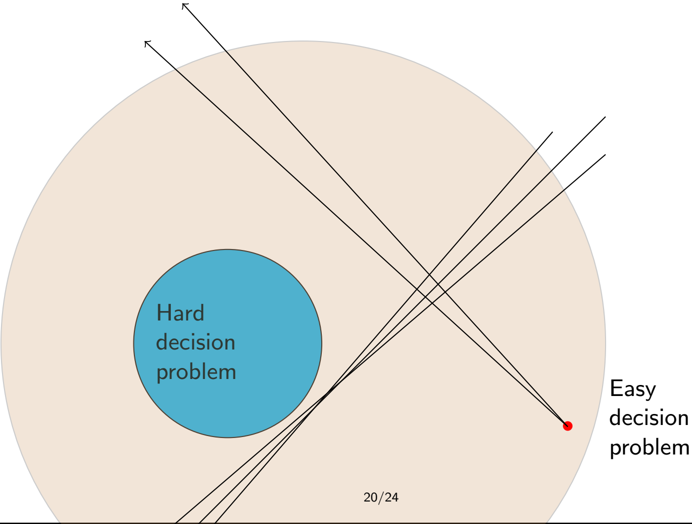

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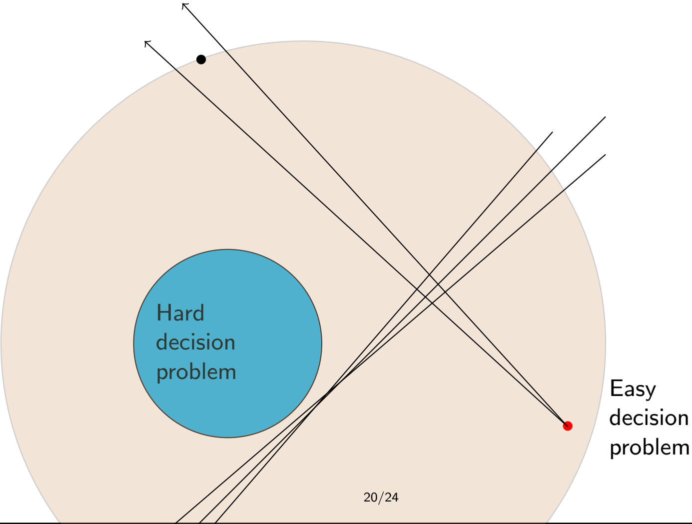

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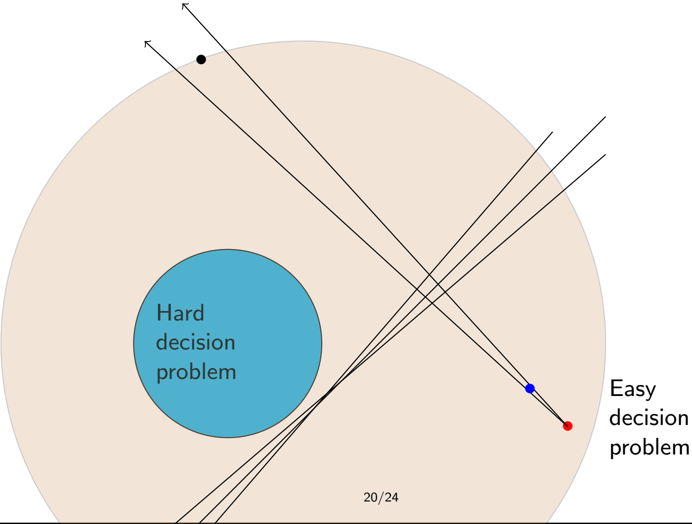

Sparsifications and Dantzig Decompositions

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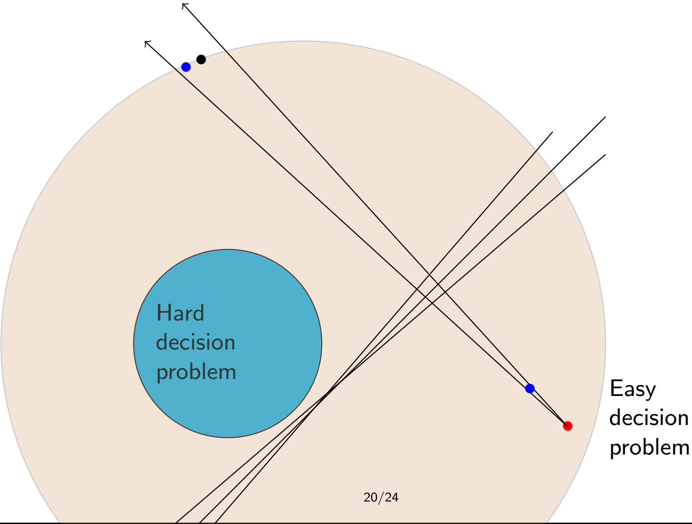

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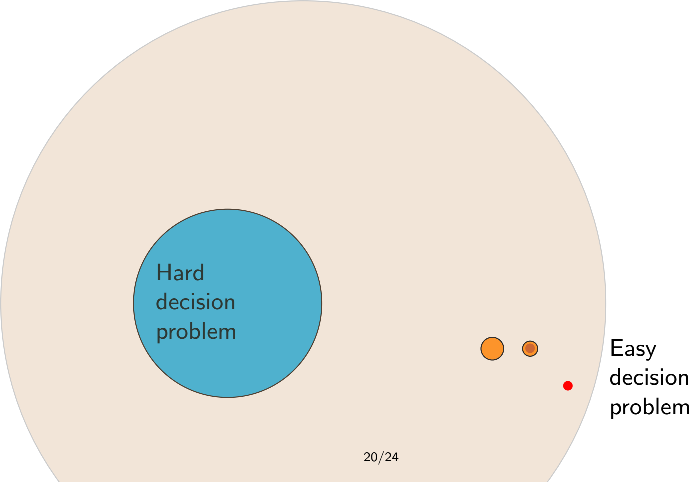

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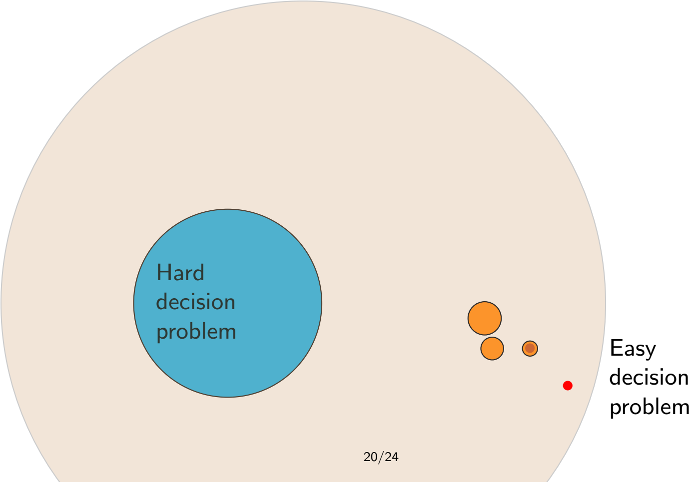

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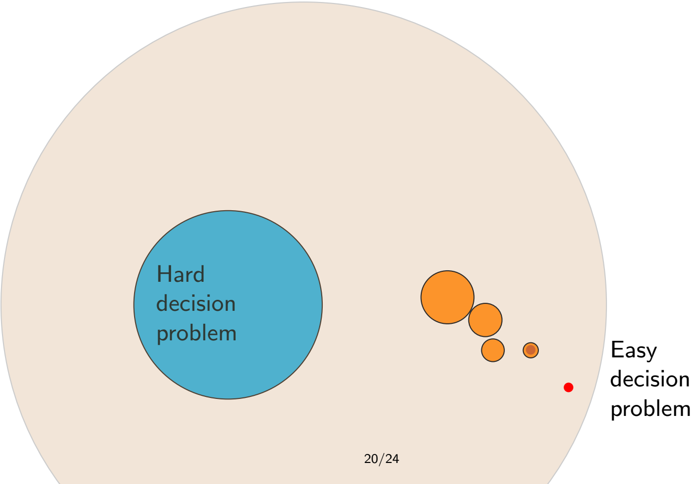

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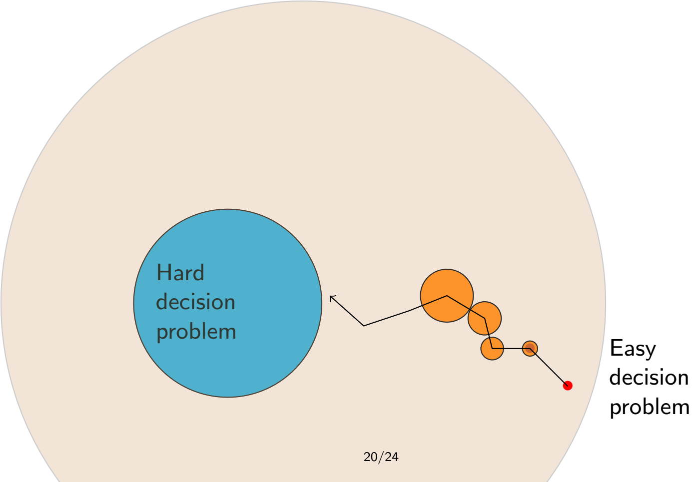
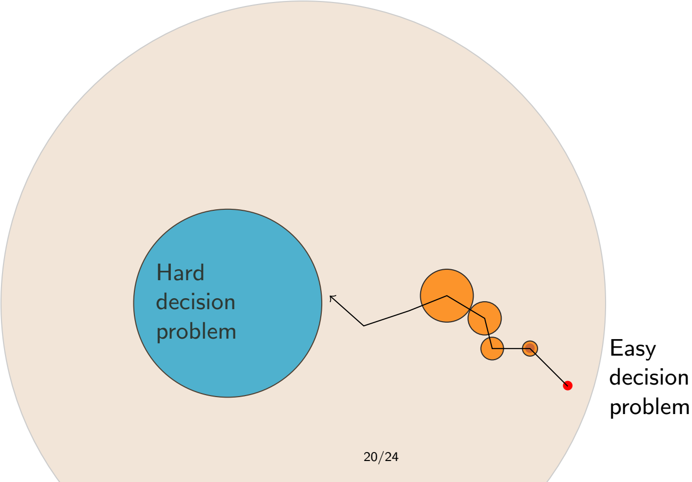


Sparsifications and Dantzig Decompositions

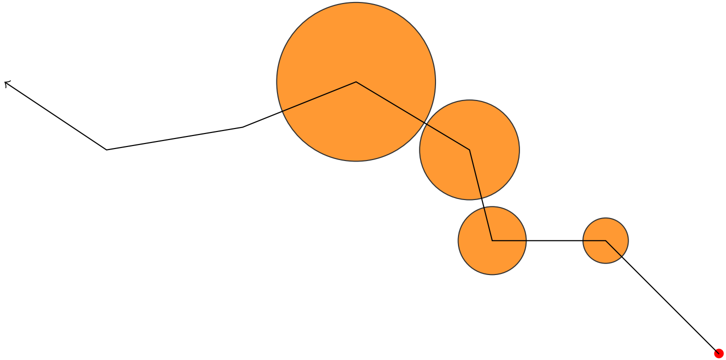
What if we sparsify $\mathbf{u}$?

Construct multiple sparsifications in parallel. Use sequentially.



Sparsify in Parallel, Use Sequentially

We saw a version of sketch in parallel, use sequentially in connectivity.
Question: Where will we be after 5 steps of MWM?
Recall: If $\mathbf{A}_i\mathbf{y} > \mathbf{b}_i$: raise $\mathbf{u}_i$, i.e., $\mathbf{u}_i \leftarrow \mathbf{u}_i(1+\epsilon)^{(\mathbf{A}_i\mathbf{y}-\mathbf{b}_i)/\mathbf{b}_i\rho}$.



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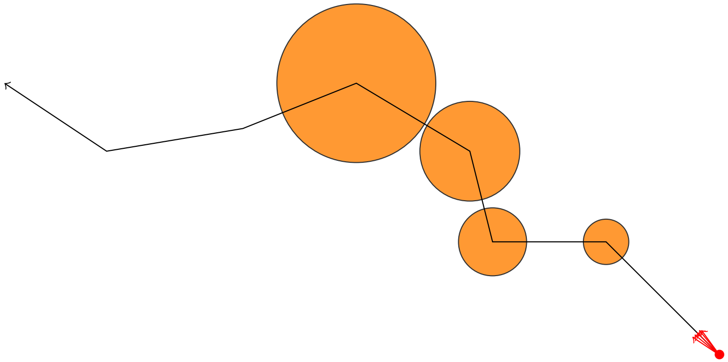
$\mathbf{u}_i(5) \in (1 \pm \epsilon)^5 \mathbf{u}_i$. Construct 5 independent sparsifications of $\mathbf{u}$.



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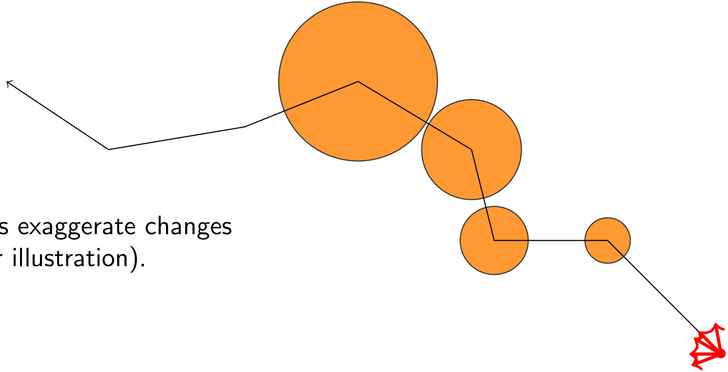

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(for illustration).

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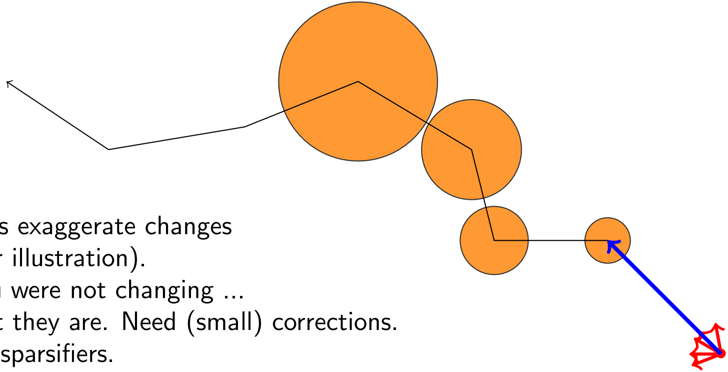

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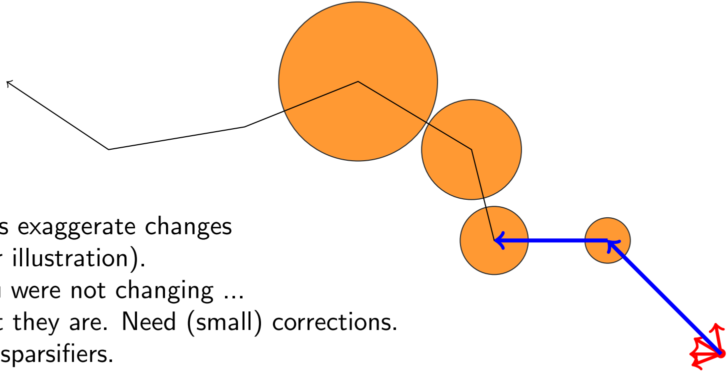

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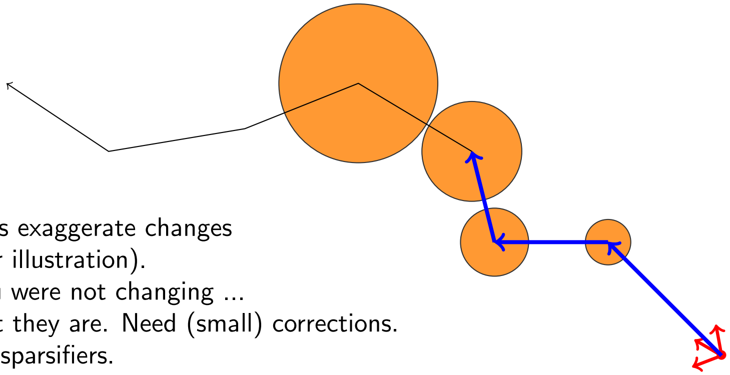

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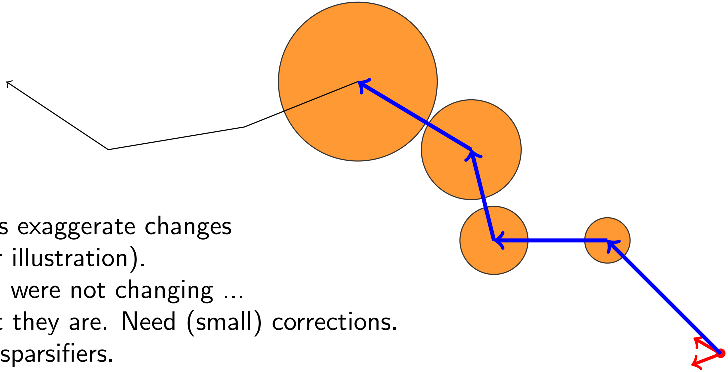

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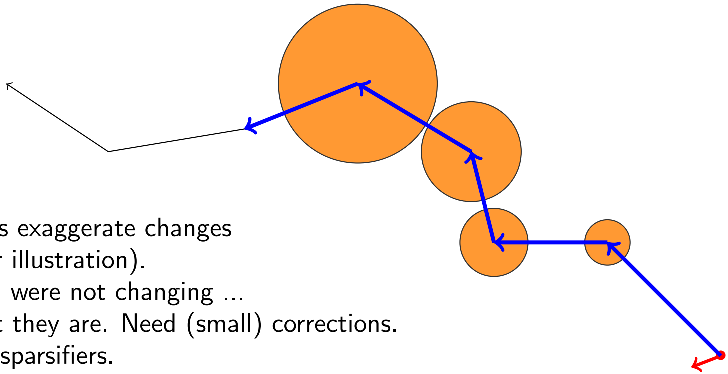

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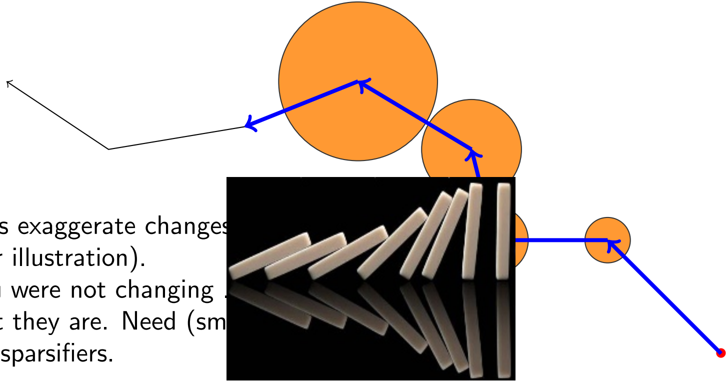

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Cuts and Sparsification

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(Again dropping $(i,j) \in E$ in the subscripts, $y_{ij} = y_{ji}$.)

$$\beta^* = \max \quad \sum_{(i,j)} w_{ij} y_{ij}$$

$$\sum_j y_{ij} \quad \leq 1 \quad \forall i$$

$$y_{ij} \quad \geq 0$$

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\sum_j y_{ij} \quad & \leq 1 \quad \forall i \quad \text{(Cut constraint!)} \\
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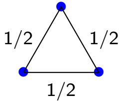
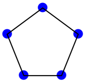
Rules out: 



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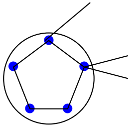
$$\sum_{i,j \in U} y_{ij} \leq \lfloor |U|/2 \rfloor \quad \Longleftrightarrow \sum_{i \in U} \left(\sum_j y_{ij} \right) - \left(\sum_{i \in U, j \notin U} y_{ij} \right) \leq 2 \lfloor |U|/2 \rfloor$$

$\sum_{i \in U, j \notin U} y_{ij} = Cut(U, V-U)$.



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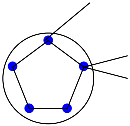
Find small cuts (with odd vertex sizes).



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Standard Algorithm: Augment, contract blossoms, ... (many rounds).
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Bipartite: ($O(\epsilon^{-2} \log n)$ rounds, no sparsification)
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. . . . . . keep best matching seen so far, ... (near) optimal

Wrap up

(1) Primitives: Sampling, Sketching and Sparsification.

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(2) LPs (MWM) on Streams.

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(9) Think differently. The real voyage of discovery ...

Thank You